Reliability of different depth-duration-frequency equations for estimating short-duration design storms

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[1] The design storm depth is defined as the rainfall depth that is expected to occur in the considered site for a given probability of occurrence and a given duration of the storm. It is generally estimated by using parametric depth-duration-frequency (DDF) equations. In particular, this article considers the frequent case when one needs to estimate the design storm depth for very short durations (typically from 5 to 45 min) while observed rainfall extremes for calibrating the DDF parameters are only available for longer durations. In this case the DDF curves are generally extrapolated below the range of durations that were used for estimating their parameters. It is well known that this procedure induces estimation errors that depend on the type of DDF equation that is used. The aim of this study is to test the capability of seven different DDF curves characterized by two or three parameters to provide an estimate of the design rainfall for storm durations shorter than 1 hour, when their parameterization is performed by using data referred to longer storms. The results point out that a proper choice of the DDF curve may improve the reliability of the design storm depth estimation significantly. In particular, DDF curves with three parameters provide a conservative estimation with an average relative error which never exceeded 20% for all the storm duration considered here.

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1. Introduction

[2] The rainfall depth-duration-frequency (DDF) curve allows one to compute the design storm depth $h^*(P, d)$, which is the expected rainfall depth for a given duration, d, of the storm and a given probability of occurrence, P [*Chow et al.*, 1988; *Chen*, 1983]. It is current practice in applied engineering to express the probability of occurrence of an event by specifying its return period, T, which is the average length of time separating the event itself from the closest one having equal or greater magnitude. Estimation of the design storm depth is required when designing artificial drainage systems or engineering works that interfere with natural river networks. When observed rainfall extremes are available for the storm duration d_i of interest, the design storm depth can be estimated by fitting on such data a suitable extreme value probability distribution, that allows one to compute the rainfall quantile $Q_i(T, d_i)$ corresponding to the return period T. In order to estimate the design storm depth for durations of the event that are different with respect to the ones the observed data refer to, the DDF parametric curve is used to interpolate the above quantiles, therefore allowing to compute $h^*(T, d)$ for any storm duration. The literature proposes many analytical expressions for the DDF parametric curves, that are characterized by a variable number of parameters, typically from 2 to 4. Increasing the number of parameters allows one to obtain a more flexible fitting of the rainfall quantiles $Q_i(T, d_i)$, but the effects of parameter uncertainty are amplified and therefore the uncertainty of the estimated $h^*(T, d)$ is amplified as well.

[3] *Burlando and Rosso* [1996] proposed an approach for limiting the parameterization requirements of DDF curves by assuming that the rainfall data, once suitably rescaled, follow the same probability distribution for any storm duration. This technique presents the considerable advantage of allowing one to express the DDF curve as an analytical function of the return period, therefore not requiring the a priori selection of T itself. This type of approach was not considered in the present study as the scale invariance assumption has been shown to be often not consistent with the statistical behaviors of short-duration extreme rainfalls (see section 7).

[4] Design storm depth estimation is complicated when one refers to short storm durations, as typically happens when designing urban drainage systems. In fact, in many countries, short-duration rainfall data are rarely available, and therefore the DDF curves are typically extrapolated below the range of durations that were used for calibrating their parameters. It is well known that such procedure amplifies the estimation error, the magnitude of which is expected to depend on the type of DDF equation that is used.

[5] The literature presents several analytical formulations for the DDF curves [*Yarnell*, 1935; *Chow*, 1964; *Bell*, 1969; *Chen*, 1983; *Garcia-Bartual and Schneider*, 2001]. The aim of this study is to test the capability of different DDF curves to estimate the design storm depth for durations of the event shorter than 1 hour, when their calibration is

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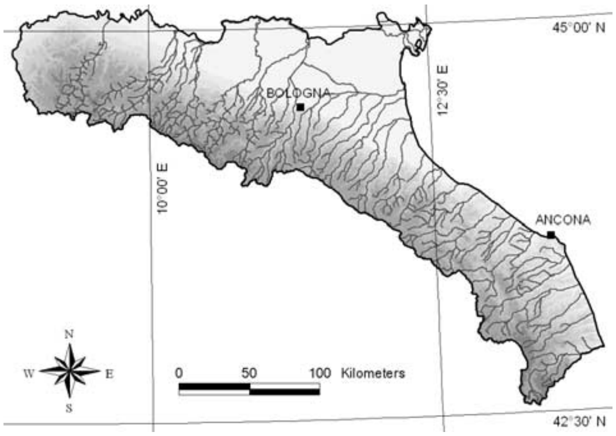

Figure 1. Study area.

performed by only using data referred to longer durations. Seven different DDF equations (4 with 2 parameters and 3 with 3 parameters) are tested against an extremely large historical database of rainfall data collected in northern Italy. Focus has been made on DDF formulations that have no more than 3 parameters as these are the most used in practice in order to limit the effects of parameter uncertainty (see section 3) [see also *Koutsoyiannis et al.*, 1998]. For the same reason, we estimated the DDF curve parameters by using rainfall data that refer to a wide range of durations d_i (from 1 to 24 hours).

2. Study Area and Rainfall Data

[6] The study region (see Figure 1) includes the administrative regions of Emilia-Romagna and Marche, in northern central Italy. The area is bounded by the Po River to the north, the Adriatic Sea to the east, and the divide of the Apennine Mountains to the southwest. The northeastern portion of the study area is predominantly flat, while the southwestern and coastal parts are mainly hilly and mountainous. The mean annual precipitation varies on the study region from about 500 to 2500 mm.

[7] The database of extreme rainfall consists of the annual series of precipitation maxima with duration d equal to 15, 20, 30 and 45 min (subhourly durations); 1, 3, 6, 12 and 24 hours (daily durations). These data were obtained for a dense network of rain gauges from the National Hydrographical Service of Italy (SIMN) in the period 1935–1989. The study considered the rain gauge stations with at least 10 years of observation for each subdaily duration and at least 2 subhourly durations, obtaining in this way a set of 131 rain gauge stations. The consistency of the selected rainfall database is summarized in Table 1.

3. Computation of the Extreme Rainfall Quantiles

[8] The first step for estimating the DDF curves is the computation of the rainfall quantiles $Q(T, d_i)$ for each rain gauge and each storm duration d_i. To this end, the Gumbel extreme value probability distribution [*Gumbel*, 1958], that is also know as extreme value type 1 distribution, was used. Accordingly to the Gumbel model, the probability of non exceedance $P(h, d_i)$ of the rainfall depth h for storm duration d_i can be expressed as:

$$P(h, d_i) = \exp\left[-\exp\left(-\frac{h - u_i}{\alpha_i}\right)\right]. \quad (1)$$

The distribution parameters u_i and a_i, referred to the storm duration d_i, have been estimated by applying the method of moments.

[9] The rainfall quantile $Q(T, d_i)$ can be computed by assuming a constant design probability of occurrence P and then inverting (1). By considering that $P = (T - 1)/T$, where T is the design return period the DDF refers to, one obtains:

$$Q(T, d_i) = u_i - \alpha_i \log\left[-\log\left(\frac{T-1}{T}\right)\right]. \quad (2)$$

A key step in order to estimate the rainfall quantiles given by (2) is the selection of the return period T the DDF curve refers to. Given that urban drainage systems are usually designed for T variable from 2 to 10 years in urban and industrial areas, a value of T = 10 years was initially adopted in this study. Subsequently, in order to inspect whether the results also apply for different return periods, the analysis was repeated for T = 2, 5, 10, 20 and 50 years.

[10] Other extreme value models could be used herein. The Gumbel distribution was preferred as it has been widely applied in the past all over the world in order to fit heavy rainfall. Nevertheless, we performed an extensive study in order to verify the goodness of fit provided by the Gumbel model for our data.

[11] A first analysis was carried out by computing the average regional value of the third and fourth-order L moments for the observed rainfall extremes [*Hosking and Wallis*, 1997]. Figure 2 shows the L moment ratio diagram [*Hosking and Wallis*, 1993] where the statistics of the observed data are compared with the ones of the Gumbel, Logistic, Exponential, Normal and Uniform distributions. One can see that the theoretical L skewness and L kurtosis values for Gumbel distribution are very close to the regional L skewness and L kurtosis values for hourly and subhourly rain gauges, therefore indicating that the Gumbel model is a suitable parent distribution for the considered data. In order to further test the suitability of the Gumbel model in each single application, the Kolmogorov-Smirnov test has been applied for each rain gauge and each storm duration herein considered. It turned out that the test was always satisfied at a 95% confidence level, therefore confirming the goodness of fit. However, even if the Gumbel model provided an

Table 1. Consistency of the Selected Database: Number *Ng* of Rain Gauges and Total Number *Ny* of Years of Observation for the Different Storm Durations

d	*Ng*	*Ny*
Subdaily durations	308	8845
45 min	40	527
30 min	173	3481
20 min	110	1640
15 min	111	1533

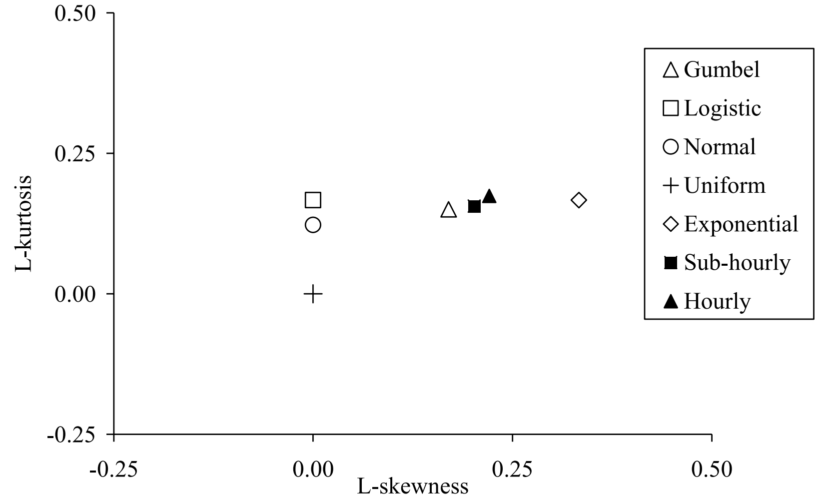

Figure 2. Diagram of L moment ratios for the application data.

acceptable fit within the present analysis, it is important to note that recent studies pointed out some limitations of such approach for the estimation of DDF curves. In particular, *Koutsoyiannis* [2004] found that the Gumbel distribution may underestimate the design rainfall depth for large return periods.

4. Testing the Reliability of the DDF Equations

[12] For each rain gauge, the seven different DDF equations considered here were fitted by using observed data referred to storm durations of 1, 3, 6, 12 and 24 hours. The least squares approach was used in order to optimize the DDF parameters. [*Chow et al.*, 1988]. The parameters have been estimated independently for each return period T. Afterward, the DDF curves were used to estimate the design storm depth $h^*(T, d)$ for storm durations equal to 15, 20, 30, and 45 min. The obtained estimates were compared to the rainfall quantiles $Q(T, d_i)$ derived by fitting the Gumbel distribution directly onto the data observed for the corresponding storm duration (direct statistical analysis).

[13] The capability of the different DDF equations to provide a reliable estimate of the short-duration design storm depth was tested by computing the relative error $\varepsilon_j(d_i)$ for each rain gauge and each storm duration, the mean absolute percentage error $E\%(d_i)$, for each duration (averaged over all the rain gauges) and the root mean square error RMSE (averaged over all the rain gauges and storm durations). The subscript j and i refer to the rain gauge and the storm duration, respectively. These goodness of fit measures are computed with the following relationships,

$$\varepsilon_j(d_i) = \frac{h_j^*(10, d_i) - h_j(10, d_i)}{h_i(10, d_i)}, \quad (3)$$

$$E\%(d_i) = \frac{100}{N} \cdot \sum_{j=1}^{N} \frac{|h_j^*(10, d_i) - h_j(10, d_i)|}{h_j(10, d_i)}, \quad (4)$$

$$RMSE = \frac{1}{M} \cdot \sum_{i=1}^{M} \sqrt{\sum_{j=1}^{N} \frac{\left(h_j^*(d_i, 10) - h_j(d_i, 10)\right)^2}{N}}, \quad (5)$$

where M and N are the number of storm durations and rain gauges stations, respectively; $h_j(T, d_i)$ and $h_j^*(T, d_i)$ represents the rainfall depth of jth station and storm duration d_i estimated through the DDF functions and direct statistical analysis, respectively.

5. Depth-Duration-Frequency Equations

[14] The literature presented several DDF equations that can be used to interpolate rainfall quantiles corresponding to different storm durations. In this study we considered 7 different formulations that are characterized by two or three

Table 2. Depth-Duration-Frequency Curves Considered in the Study, Number of Their Parameters, and Estimates of the Rainfall Intensity When the Duration d Approaches Zero[a]

DDF	Function ($h^*(d)$ =)	Number Parameter	Intensity for $d \to 0$
1	$a\, d^b$	2	∞
2	$a \cdot (b + d)^{-1} \cdot d$	2	a/b
3	$(a - b \cdot \ln d) \cdot d$	2	∞
4	$a \cdot b^{(28^{0.1} - d^{0.1}) \cdot 2.5} \cdot d$	2	$a \cdot b^{3.489}$
5	$a \cdot (d + c)^{-b} \cdot d$	3	a/c^b
6	$a \cdot (d^b + c)^{-1} \cdot d$	3	a/c
7	$(c + a \cdot (b + d)^{-1}) \cdot d$	3	$c + a/b$

[a]To simplify the notation, the dependence on the return period T of the parameters a, b, and c is omitted.

Table 3. Interpolation Error: Mean Absolute Percentage Error E Averaged on All the Hourly Durations

Function	E, %
DDF1	1.84
DDF2	9.08
DDF3	17.47
DDF4	4.25
DDF5	1.74
DDF6	1.77
DDF7	1.93

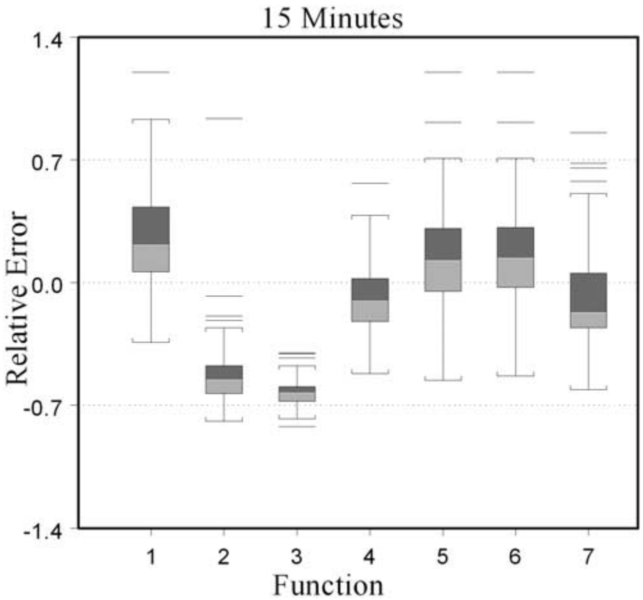

Figure 3. Box plots of the relative error $\varepsilon_j(d_i)$, d_i = 15 min.

parameters. These latter will be denoted by the symbols a, b and c. The DDF equations are summarized in Table 2, along with the value of the rainfall intensity estimated by each of them when the storm duration d approaches 0. The parameters a, b and c were estimated using the least squares method. Details about practical applications of these curves are given by *Bernard* [1932], *Linsley et al.* [1949], *Garcia-Bartual and Schneider* [2001], *Témez* [1978], *Koutsoyiannis et al.* [1998] and *Keers and Wescott* [1977].

[15] Some important considerations can be drawn about the DDF curves considered here.

[16] 1. DDF 1 (also known as Montana Curve) is probably the most used in real world applications. This curve provides estimates of the rainfall intensities tending to infinity when d approaches 0.

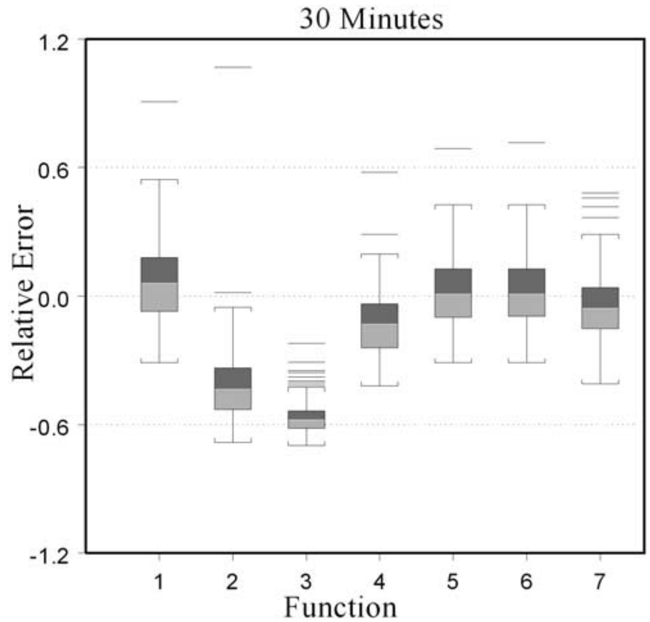

Figure 5. Box plots of the relative error $\varepsilon_j(d_i)$, d_i = 30 min.

[17] 2. DDF 1 and DDF 2 can be regarded as special cases of either DDF 5 and DDF 6. Moreover DDF 7 reduces to DDF 2 by setting c = 0.

[18] 3. DDF 5 and DDF 6 are themselves special cases of the more general four-parameter depth-duration-frequency function considered by *Koutsoyiannis et al.* [1998], who observed that the four-parameter expression can be over-parameterized in practical applications, in consideration of the consistency of the databases usually available. Therefore we limited our attention to DDF that have at most three parameters.

[19] 4. DDF 3 is valid for a given range of d only, as it provides negative rainfall depths when $a < b \ln d$.

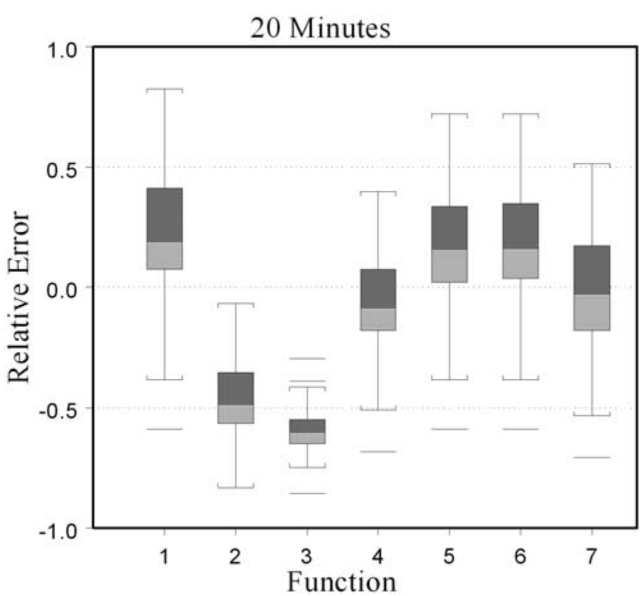

Figure 4. Box plots of the relative error $\varepsilon_j(d_i)$, d_i = 20 min.

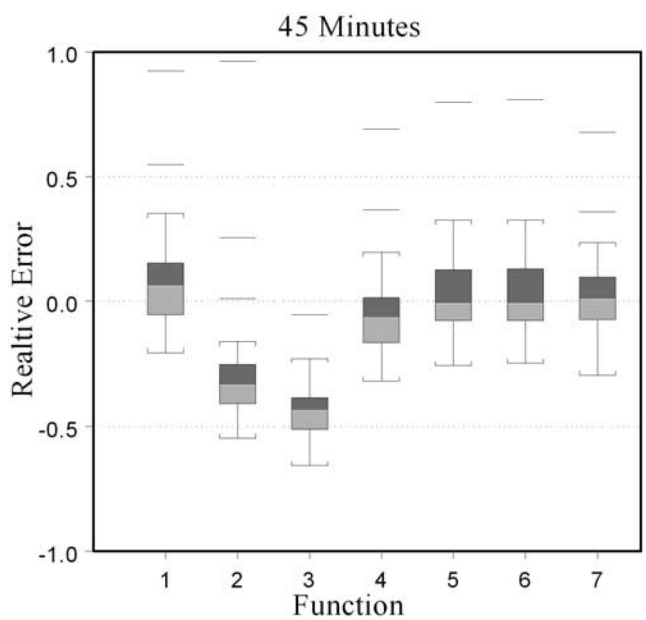

Figure 6. Box plots of the relative error $\varepsilon_j(d_i)$, d_i = 45 min.

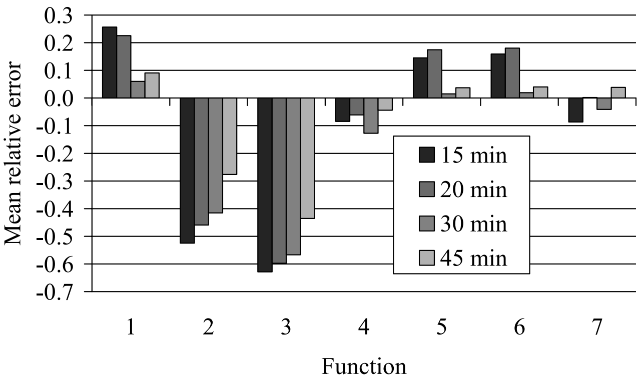

Figure 7. Mean relative error $\overline{\varepsilon}$(di), for each subhourly duration, averaged over all the rain gauges.

[20] 5. DDF 4 was proposed by the Spanish Water Authority Centro de Estudios Hidrograficos – CEDEX [*Témez*, 1978] and is extensive used for practical hydrological purposes all over Spain [*Garcia-Bartual and Schneider*, 2001].

[21] 6. DDF 7 is the only function that yields nonzero intensity for duration tending to infinity.

6. Results

[22] Before commenting on the results, it is interesting to analyze the efficiency of the different DDF curves in interpolating the considered rainfall quantiles that refer to storm durations ranging from 1 to 24 hours. Table 3 shows the interpolation errors in terms of mean absolute percentage error averaged on all rain gauges and durations (E%). One can see that the smaller interpolation errors are given by DDF1, DDF 5, DDF 6 and DDF7, which give $E\% < 2\%$. The worst performances are obtained with DDF 3, for which $E\% = 17.47\%$.

[23] Figures 3–6 show the box plots of the relative error in the estimation of the design storm depth for each subhourly rainfall duration (15, 20, 30, and 45 min). The box plots display the three quartiles ($\varepsilon_{0.25}$, $\varepsilon_{0.5}$ and $\varepsilon_{0.75}$) on the rectangular box and also show two vertical segments connecting the extreme of the rectangle with the values $\varepsilon_{0.25} - 1.5(\varepsilon_{0.75} - \varepsilon_{0.25})$ and $\varepsilon_{0.75} + 1.5(\varepsilon_{0.75} - \varepsilon_{0.25})$. The outliers are represented with horizontal segments.

[24] Figure 7 presents the relative error, ε_j (d_i), averaged over all the rain gauges. Figures 8 and 9 show the mean absolute percentage error, $E\%(d_i)$, calculated on all the stations, for durations d_i equal to 15, 20, 30 and 45 min (Figure 8) and averaged on all subhourly durations (Figure 9). Finally the RMSE values for each DDF curve, averaged on all subhourly durations, are reported in Table 4.

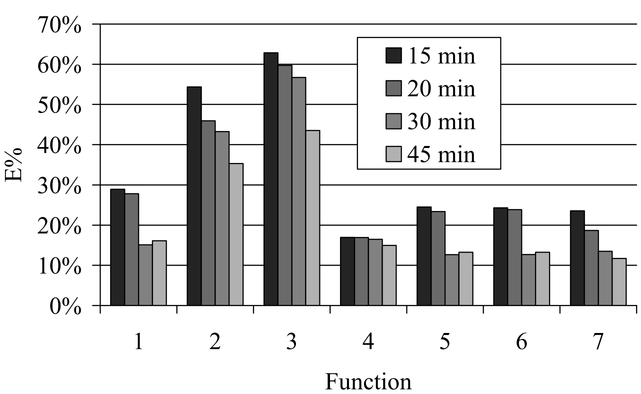

Figure 8. Mean absolute percentage error $E\%(d_i)$ for each duration shorter than 1 hour.

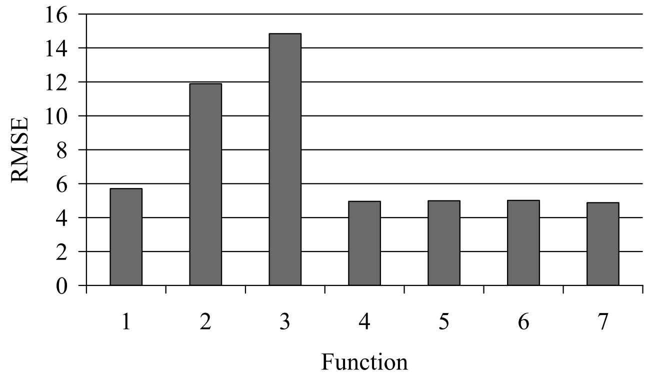

Figure 9. Mean absolute percentage error $E\%(d_i)$ averaged on all the subhourly durations, T = 10 years.

[25] By analyzing the goodness of fit measures one can first of all observe that the DDF 1 significantly overestimates the rainfall depth for short storm durations. This results is already known among practitioners and is explained by the analytical form of DDF 1. As a matter of fact, such formulation gives rainfall intensities tending to infinity as d approaches 0 (see Table 2). In detail, the plot of the mean relative error (Figure 7) and the box plots (Figures 3–6) show a significant overestimation that, as expected, increases with decreasing storm duration.

[26] By analyzing the performances of the other DDF formulations one can see that DDF 2 and DDF 3 tend to underestimate the design storm depth and therefore their use is not advisable for practical applications. DDF 4 and DDF 7 give the smallest errors, and therefore deserve a more refined analysis of their performances. In detail, DDF 4, which is a two-parameter equation, provides interesting results, the mean absolute percentage errors $E\%(d_i)$ being less than 20% for all storm durations. This picture of its performances would make DDF 4 interesting for real world applications. However the box plots and the Figure 7 show that the rainfall depth is underestimated on average. Of course this behavior is not appreciated by practitioners, as the estimation of the design storm depth given by DDF 4 may be not conservative. DDF 7 also tends to underestimate the design storm depth for durations d equal to 15 and 30 min (see Figure 7).

[27] Taking into account this latter consideration, one can conclude that DDF 5 and DDF 6 give the most interesting

Table 4. Extrapolation Error: Root-Mean-Square Error Averaged on All the Subhourly Durations

Function	RMSE, mm
DDF1	5.71
DDF2	11.89
DDF3	14.84
DDF4	4.96
DDF5	4.99
DDF6	5.01
DDF7	4.88

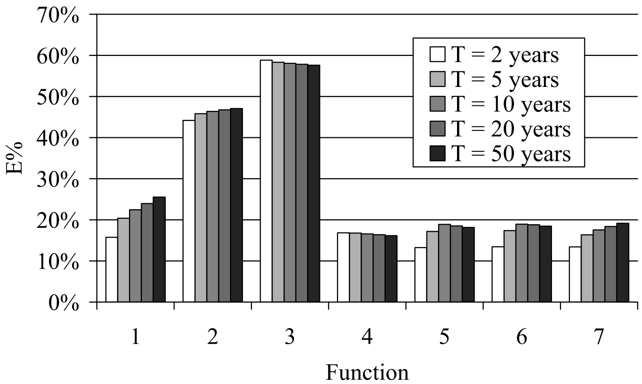

Figure 10. Mean absolute percentage error $E\%(d_i)$ averaged on all the subhourly durations for different return periods.

results, as they provide a reasonably reliable and conservative evaluation of the design storm depth. In fact, the mean relative error is positive on average for all the storm durations and the mean absolute percentage error is around 20%.

[28] Focusing on the other return periods, Figure 10 shows the mean absolute percentage error, $E\%(d_i)$, averaged on all the subhourly durations for different return periods (T = 2, 5, 10, 20, 50 years). One can see that the results are similar. It is interesting to note that DDF 5 and DDF 6 provide a good performance for all return periods.

7. Discussion and Conclusion

[29] The results shown in the previous section have highlighted the limitations of the widely used DDF 1 curve for describing the behaviors of short-duration storms. This result has a physical justification. In fact, under the assumption that the parameter b is independent of the return period (a condition which is frequently not violated in practice), the validity of DDF 1 implies the satisfaction of the scale invariance assumption by *Burlando and Rosso* [1996], which is expressed through the relationship

$$h(\lambda d) \sim \lambda^b \cdot h(d), \tag{6}$$

where $\lambda \in \Re$ and $\sim$ indicates equality in the probability distribution. Equation (6) conveys that the rainfall depth, once rescaled through a suitable power law multiplier, follows the same probability distribution regardless of storm duration. However, many authors already pointed out that this type of scale invariance assumption is frequently violated in practice, as rainfall data typically show a transition in their scaling behavior for storm duration of about 1 hour [*Olsson and Burlando*, 2002; *Marani*, 2003]. In particular, *Marani* [2003] shows that the variance of the rainfall data decreases more rapidly when referring to short durations with respect to what is observed for longer storms. This result explains from a physical point of view the overestimation of the design storm depth made by DDF 1 and therefore its inability to summarize the statistical behavior of point rainfall for any storm duration. Therefore a different DDF equation is required, which should better fit the transition in the scaling regime of rainfall.

[30] Of course, dealing with empirical relationships, the choice of the best performing DDF equation depends on the capability of the different formulations to infer the local climatic behaviors. On the basis of this study, one can argue that DDF 5 and DDF 6 provide the most interesting outcomes for the geographical location considered here. It is interesting to note that the three-parameter function DDF 5 has been recently given a physical explanation based on maximum entropy considerations [*Koutsoyiannis*, 2006]. In this respect, we believe that further analyses about design storm depth estimation for short duration events could be carried out through a better understanding of the physical processes. To this end, regional studies could also be a viable tool to better exploit the available hydrological information.

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