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Regional flow-duration curves: reliability for ungauged basins

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Abstract

A flow-duration curve (FDC) illustrates the relationship between the frequency and magnitude of streamflow. Applications of FDC are of interest for many hydrological problems related to hydropower generation, river and reservoir sedimentation, water quality assessment, water-use assessment, water allocation and habitat suitability. This study addresses the problem of FDC estimation for ungauged river basins, assessing the effectiveness and reliability of several regional approaches. The study refers to a wide region of eastern central Italy and adopts a jack-knife cross-validation procedure to evaluate the uncertainty of regional FDC's, comparing it with the uncertainty of empirical FDC's constructed from short samples of streamflow data. The results (a) provide an evaluation of the reliability of the regional FDC's for ungauged sites, (b) show that the reliability of the three best performing regional models are similar to one another, and (c) demonstrate that empirical FDC's based on limited data samples generally provide a better fit of the long-term FDC's than regional FDC's.

Keywords: Streamflow regime; Regional analysis; Jack-knife cross-validation; Eastern-central Italy

1. Introduction

A flow-duration curve (FDC) provides the percentage of time (duration) a daily or monthly (or some other time interval) streamflow is exceeded over a historical period for a particular river basin (see e.g., [28]). FDC may also be viewed as the complement of the cumulative distribution function of the considered streamflows (e.g., [15,28]). The earliest use of FDC's is attributed to Clemens Herschel and dates back to 1880 [10]. Nevertheless, FDC's are still widely used by hydrologists around the world in numerous water related applications like hydropower generation and planning and design of irrigation systems [10,22], management of stream-pollution (see for example [22]), river and reservoir sedimentation and fluvial erosion (see for example [6,10,21,31]). Vogel and Fennessey [29] present a comprehensive review of FDC applications in water resources planning and management.

Empirical FDC's can be easily constructed from streamflow observations using standardised non-parametric procedures described for example by Vogel and Fennessey [28]. Unfortunately the scarcity of streamflow data is a common problem, as shown by the large number of studies addressing the regionalisation of FDC's for different geographic regions around the world, as Canada [16], Greece [17], India [23], Italy [5,11], Taiwan [32], Philippines [19], Portugal [7], South Africa [25] and

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United States [9]. The regionalisation of FDC's appears therefore to be an essential operative tool when dealing with ungauged river basins or short streamflow records.

Nonetheless, the literature on the regionalisation of FDC's is still sparse, if compared to the vast number of scientific papers addressing the problem of regional flood frequency analysis (see e.g., [20]). Also, there is a generalised tendency to propose regional models and recommend their application to ungauged sites without an appropriate validation of the estimated FDC's. To our knowledge, the study by Yu et al. [32] represents the only exception. The study derives regional FDC's for a rather small geographical area (3155 km^2) and uses a bootstrap resampling procedure to validate the regional curves.

Our study considers a wide geographical region in eastern central Italy, develops several regional models of daily streamflow FDC's and analyses the performances of the models. The study has two main goals: (a) to quantify the reliability and effectiveness of different regional procedures through the application of a jackknife cross-validation [3], (b) to compare the reliability of the regional FDC's with the reliability of the empirical FDC's constructed using short streamflow records (record length equal to 1, 2 and 5 years).

2. Construction of flow-duration curves

FDC is probably one of the most informative methods of displaying the complete range of river discharges, from low flows to flood events [24]. Its traditional interpretation is reported in the scientific literature as period-of-record FDC (e.g., [24,28]), from now on simply referred to as FDC, which consists of the complement of the cumulative distribution function of the daily (or some other time interval of) streamflows over the whole available period of record.

The construction of a FDC using the streamflow observations can be performed through non-parametric procedures consisting of two main steps: (a) the observed streamflows q_i, $i = 1, 2, \ldots, N$, are ranked to produce a set of ordered streamflows $q_{(i)}$, $i = 1, 2, \ldots, N$, where N is the sample length, and $q_{(1)}$ and $q_{(N)}$ are the largest and the smallest observations, respectively; (b) each ordered observation $q_{(i)}$ is then plotted against its corresponding duration D_i, which is generally dimensionless and coincides with an estimate, p_i, of the exceedance probability of $q_{(i)}$. If the Weibull plotting position (WPP) is used, p_i reads

$$p_i = P(Q > q_{(i)}) = \frac{i}{N+1} \tag{1}$$

A different approach is described in LeBoutillier and Waylen [15] and Vogel and Fennessey [28] who proposed an annual interpretation of flow-duration curves. This interpretation considers FDC's for individual years (AFDC's), each one constructed analogously to the FDC, using only the hydrometric information collected in a calendar or water year. Vogel and Fennessey [28] illustrate how to derive for gauged river basins (a) the median AFDC, which represents the distribution of streamflows in a median hypothetical year and is not affected by the observation of abnormally wet or dry periods during the period of record, (b) the confidence intervals around the median FDC, summarising the observed inter-annual variability of streamflows, (c) the AFDC associated with a given recurrence interval.

AFDC's were shown to be less sensitive to the record period than traditional FDC, especially in the area of low flows [28], and can be effectively employed in deriving flood and low flow indexes, which are usually determined from the probabilistic structure of daily or weekly flows (see e.g. [5]). Nevertheless, FDC's are still widely used in hydrological practice [11,23,25] and, unlike AFDC's, can be effectively used for filling gaps and extending streamflow series, and, when a regional FDC model is available, for generating synthetic streamflow series at ungauged river basins (see e.g., [8,14,25]).

3. Regionalisation of flow-duration curves: a review

As previously outlined, the lack of streamgauges and the limited amount of streamflow observations characterise several geographical areas around the world. This condition, along with the worldwide pursuit of the optimal estimation and management of water resource, led to the formulation and proposal of numerous procedures for regionalising FDC, whose common objective is the estimation of FDC's at ungauged river basins or the enhancement of empirical FDC's constructed for streamgauges where only a limited amount of hydrometric information is available.

A rough classification of the available regionalisation procedures into two broad categories distinguishes between procedures that view FDC as the complement of the cumulative frequency distribution and procedures that do not make any connection between FDC and the probability theory.

The procedures belonging to the first category use stochastic models to represent FDC's [15,16] and are generally applied as follows: (a) a suitable frequency distribution is chosen as the parent distribution for a particular region; (b) the distribution parameters are estimated on a local basis for the gauged river basins located in the study region using the streamflow observations; (c) regional regression models are then identified for predicting the distribution parameters at ungauged basins on the basis of the geo-morphological and climatic characteristics of the basins. These procedures will be referred hereafter to as statistical approaches.

Concerning the second category, a further division into two different subsets can be envisaged. The first subset includes all procedures whose basic idea is the representation of the FDC's by analytical relationships. The parameters of the relationships for ungauged river basins are then estimated through regional models, similarly to the parameters of the parent distribution for the statistical approaches. The second subset comprises regional procedures that use standardised graphical representations of FDC's with a regional validity instead of analytical relationships. In order to differentiate between the two subclasses of non-statistical procedures, the remainder of the paper will refer to parametric and graphical approaches.

3.1. Statistical approaches

A regional hydrological model for estimating FDC's of daily streamflows at ungauged river basins was proposed by Fennessey and Vogel [9]. The authors, as originally suggested by Beard [1], adopt the two-parameter lognormal frequency distribution to represent the distribution of daily streamflows over the interval $0.50 \leqslant p \leqslant 0.99$. The exceedance probability reads in this case,

$$p = P(Z > z_p) = 1 - (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{z_p} \exp\left(-\frac{1}{2}x^2\right) \mathrm{d}x \qquad (2a)$$

where,

$$z_p = [\ln(q_p) - \mu]/\sigma \qquad (2b)$$

is the pth percentile of the zero mean and unit variance normally distributed random variable Z, whereas q_p is the pth percentile of the lognormally distributed random variable with parameters μ and σ. The authors analyse a large number of catchments in Massachusetts and estimate μ and σ by minimising within the range $0.50 \leqslant p \leqslant 0.99$ the sum of the squared residuals between the exceedance probabilities of the observations estimated through (1) and through the stochastic model (2). Using the hydrometric and geomorphoclimatic information available for the gauged river basins the authors identify through multivariate regression analyses two regional models to estimate μ and σ at ungauged basins of the study region.

LeBoutillier and Waylen [16] propose a stochastic approach for the regionalisation of the mean AFDC of daily streamflows for the entire Province of British Columbia, Canada. The authors employ the five-parameter mixed lognormal distribution as the parent distribution and utilise cluster analysis to identify groupings of basins having similar streamflow regimes.

Claps and Fiorentino [5] present a statistical approach to the regionalisation of AFDC's. The authors consider 14 river basins located in southern Italy and, for each basin, fit a two-parameter lognormal distribution to each empirical annual FDC's. The authors show for each site that the two series of annual parameters are normally distributed, and propose regional models to estimate the parameters of the normal distributions for ungauged sites.

Singh et al. [23] present a statistical regional model for the estimation of FDC's for ungauged river basins in the Himalayan region. The authors adopt the normal frequency distribution to represent the standardised and normalised 10-day streamflow series observed in the study region. The standardisation of each series is performed by dividing the observations by the mean annual flow of the corresponding streamgauge, while the normalisation is obtained through a power transformation of the data [2]. The estimation of the mean annual flow can be performed for ungauged sites through a regional model as a function of the basin area.

Croker et al. [7] propose a theoretical framework to address the construction of FDC's for ephemeral river basins using the theory of total probability. The authors present a regional model to estimate the probability of zero/non-zero flows for Portuguese ungauged river basins. The model estimates the probability of zero-flows as a function of the mean annual precipitation, as a surrogate of both geographical location and climatic conditions, allowing the derivation of FDC's for ungauged river basins, either ephemeral or perennial.

3.2. Parametric approaches

Quimpo et al. [19] propose a regional parametric approach for estimating FDC's at ungauged potential small hydropower sites in the Philippines. The authors suggest the following two-parameter exponential equation to represent FDC's of daily streamflows:

$$Q(D) = Q_A \exp(-cD) \qquad (3)$$

where $Q(D)$ indicates the daily streamflow associated to duration D, and Q_A and c are the parameters of the equation. In order to evaluate FDC's at ungauged sites, the authors identify a regional regression model that estimates Q_A as a function of the drainage area and provide a contour map representation of c for the whole archipelago of the Philippines.

Mimikou and Kaemaki [17] perform a study analogous to the study by Quimpo et al. [19]. The authors consider several analytical equations, (3) included, to represent the monthly FDC's observed in north-western Greece, and suggest to use a third order polynomial equation to estimate $Q(D)$ as

$$Q(D) = a - bD + cD^2 - dD^3 \qquad (4)$$

where the parameters a, b, c and d have to be non-negative. In order to estimate the FDC's at ungauged locations the authors propose four regional regression models expressing the equation parameters as functions of the mean annual precipitation, the drainage area, the hypsometric fall and the length of the main river course.

Franchini and Suppo [11] propose a different parametric approach for estimating daily FDC's in a wide region of southern-central Italy. The authors suggested to describe the lower portion of FDC's of daily streamflows (i.e., $D \geqslant 0.3$) by adopting the following three-parameter analytical equation:

$$Q(D) = c + a(1.0 - D)^b \quad (5)$$

where the parameters a, b and c are all greater than zero. Parameter b is associated with the physical characteristics of the considered basin (e.g., imperviousness, size, etc.) and controls the concavity of the FDC (i.e., downward concavity for when $b \in]0,1[$, upward concavity if $b \in]1,+\infty[$). The estimation of the equation parameters is performed by forcing (5) to honour three points identified by three different durations and the corresponding values of daily streamflow, $[D_i, Q(D_i)]$, with $i \in [1,2,3]$. The authors present three regional regression models to estimate the values of $Q(D_i)$, with $i \in [1,2,3]$, as functions of relevant geomorphoclimatic characteristics.

Yu et al. [32] analyze 15 river basins in Taiwan and identify regional models to estimate the daily discharge Q_p, corresponding to the percentiles $p = 10, 20, \ldots, 90\%$, as functions of several geomorphoclimatic indexes. The regional models of Q_p's are then used to construct the FDC's at ungauged sites. Yu et al. [32] assess the reliability of the proposed approach at all 15 sites and determine the confidence intervals for estimated FDC's by using a bootstrap cross-validation.

3.3. Graphical approaches

Several regional procedures for the estimation of FDC's at ungauged river basins make use of graphical devices such as standardised curves (e.g., [12,13,27,25]). This study considers only the approach proposed by Smakhtin et al. [25] for the estimation of FDC's of daily streamflows in a western-central region of South Africa. The regional approach requires the following major steps: (a) standardise the FDC's for all gauged river basins by dividing the empirical FDC's by an index streamflow; the authors adopt as index streamflow the long-term mean daily flow (i.e., average of all daily streamflows in the available record period); (b) a graphical regional dimensionless FDC is then obtained by averaging the standardised empirical FDC's of all gauged river basins in the study region. The FDC for any ungauged river basin located within the study area can be estimated as the product of the dimensionless regional FDC and an estimate of the index streamflow. The authors propose two different regional models of the index streamflow for ungauged sites located in the South-African study region.

Smakhtin et al. [25] consider the entire study area as single homogeneous region, given the strong similarities existing among the empirical standardised FDC's. However, the graphical approach can also be applied to heterogeneous areas by identifying groups of basins with significant affinities in the hydrological characteristics and streamflow regimes [4].

4. Study overview

Our study focuses on the estimation of daily FDC's. Traditional period-of-record flow-duration curves (FDC's) are commonly used in Italy, and unlike AFDC's can be used directly for filling gaps and for extending daily streamflow series, or generating streamflow series at ungauged river basins (see e.g., [8,14]). In particular, hydropower engineering motivates our study, and therefore we do not consider the top end of the FDC. Instead we refer to the duration range $D \in [0.30, 0.99]$. The study analyses the statistical procedures by Fennessey and Vogel [9] and Singh et al. [23], the parametric procedures by Quimpo et al. [19], Mimikou and Kaemaki [17] and Franchini and Suppo [11] and the graphical procedure by Smakhtin et al. [25].

The analysis applies and compares the procedures as reported in the cited studies, as well as several modifications of the original procedures (see Section 6), aiming at identifying the procedure that offers the best trade-off between an accurate representation of the physical processes controlling the streamflow regime and the overall simplicity of the regional model (e.g., number of parameters and amount of information required for the application). The analysis of each regional procedure consisted of three main steps: (a) implementation of the regional model using the available daily streamflow observations; (b) analysis of the goodness-of-fit of the regional FDC's estimated with the model for a first assessment of the suitability of the procedure to the study region; (c) cross-validation of the regional model through a jack-knife resampling procedure (see e.g., [26,3]).

The second objective of the study is to compare the regional FDC's with the empirical FDC's constructed using short streamflow records (i.e., record length equal to 1, 2 and 5 years). This part of the analysis aims at identifying the minimum record length for which empirical FDC's reproduce long-term (i.e., period-of-record) FDC's more accurately than the best performing regional models. The results of this analysis might guide us through the selection between empirical and regional FDC's for gauged river basins with a limited amount of hydrometric information.

4.1. Jack-knife cross-validation

Although the specific application of the jack-knife validation varies depending on the considered regional model, the main features of the validation procedure can be summarised as follows:

1. the attention is focused on the N streamgauges available for the study area;
2. one of these gauging stations, say station s, is removed from the set;
3. the identification of the regional model (e.g. model parameterisation for a parametric approach; determination of the regional dimensionless FDC for the graphical approach; etc.) is carried out by considering the streamflow data and geomorphoclimatic characteristics of the remaining $N-1$ gauged sites;
4. using the regional model identified at step 3 the FDC for station s is estimated;
5. steps 2–4 are repeated $N-1$ times, considering in turn one of the remaining streamgauges.

The N empirical FDC's are then compared with the corresponding FDC's resulting from the model validation, hereafter referred to as jack-knifed FDC's. The comparison allows us to draw indications on the robustness and reliability of the regional model [3].

4.2. Indexes of reliability

The comparison between the empirical and jack-knifed FDC's is performed by means of several statistical indexes. In particular, the relative error, $\varepsilon_{s,j}$, for site s and duration j can be computed as

$$\varepsilon_{s,j} = \frac{\hat{q}_{s,j} - q_{s,j}}{q_{s,j}} \tag{6}$$

where $q_{s,j}$ and $\hat{q}_{s,j}$ indicate the empirical and jack-knifed daily streamflows associated with duration j. From the $\varepsilon_{s,j}$ values the mean relative error $\bar{\varepsilon}_s$ and its standard deviation $\sigma_{\varepsilon,s}$ were then calculated as

$$\bar{\varepsilon}_s = \frac{1}{N_{\mathrm{D}}^*} \sum_{j=1}^{N_{\mathrm{D}}} w_j \varepsilon_{s,j} \tag{7}$$

$$\sigma_{\varepsilon,s} = \sqrt{\frac{1}{N_{\mathrm{D}}^* - 1} \sum_{j=1}^{N_{\mathrm{D}}} w_j (\varepsilon_{s,j} - \bar{\varepsilon}_s)^2} \tag{8}$$

where N_{D} represents the number of durations considered for the comparison, $N_{\mathrm{D}}^* \leqslant N_{\mathrm{D}}$ indicates the number of durations j satisfying the condition $q_{s,j} \geqslant q_*$, and w_j a coefficient equal to 1 if $q_{s,j} \geqslant q_*$ and 0 otherwise (i.e., $\sum_{j=1}^{N_{\mathrm{D}}} w_j = N_{\mathrm{D}}^*$). q_* represents a minimum streamflow value (i.e., $q_* = 0.1\,\mathrm{m^3/s}$) below which the utilisation of the water resource is unfeasible or impacts significantly on the environment. This constraint removes the irrelevant portion of the FDC during the computation of $\bar{\varepsilon}_s$ and $\sigma_{\varepsilon,s}$.

The average of the N values of (7), $\bar{\varepsilon}$, and (8), σ_ε, with N equal to the number of sites, were calculated and used to describe the overall reliability of the regional model over the study region. Furthermore, the mean and median of the distribution of the N relative errors $\varepsilon_{i,j}$ for duration j and the $100(\alpha/2)\%$ and $100[1-(\alpha/2)]\%$ percentiles identifying the interval about the median containing the $100(1-\alpha)\%$ of the N relative errors were computed for $\alpha = 0.25$ and 0.10 (i.e., 75% and 90% error bands). Then, mean, median, and percentiles were plotted against duration to produce a diagram for the evaluation of the uncertainty of the jack-knifed FDC's of a particular regional model.

The mean relative error was computed as

$$\bar{\varepsilon}_j = \frac{1}{N^*} \sum_{i=1}^{N} w_i \varepsilon_{i,j} \tag{9}$$

where $N^* \leqslant N$ represents the number of sites with $q_{s,j} \geqslant q_*$ and w_i is a coefficient equal to 1 if $q_{s,j} \geqslant q_*$ and 0 otherwise (i.e., $\sum_{i=1}^{N} w_i = N^*$).

Finally, the following performance index was computed for each station $s = 1, 2, \ldots, N$:

$$E_s = 1 - \frac{\sum_{j=1}^{N_{\mathrm{D}}} \left(\hat{q}_{s,j} - q_{s,j}\right)^2}{\sum_{j=1}^{N_{\mathrm{D}}} \left(q_{s,j} - \sum_{j=1}^{N_{\mathrm{D}}} (q_{s,j})\right)^2} \tag{10}$$

where N_{D}, $q_{s,j}$ and $\hat{q}_{s,j}$ have the meaning illustrated before. The index defined in (10) is analogous to the Nash-Sutcliffe [18] efficiency criterion and varies between 1, perfect fit, and $-\infty$. The values of E_s, computed for $s = 1, 2, \ldots, N$, were used to calculate three further descriptors of the overall quality of the jack-knifed FDC's, P_1, P_2 and P_3, defined as the percentages of cases, over the N possibilities, for which $E_s > 0.75$ (P_1, good to fair fit), $0.75 \geqslant E_s > 0.50$ (P_2, fair to poor fit) and $E_s \leqslant 0.50$ (P_3, poor fit).

5. Study region

Fig. 1 illustrates the study region, covering an area of $17{,}830\,\mathrm{km^2}$ and 51 unregulated river basins, which are characterised by the absence of diversions, direct water abstractions, reservoirs, etc. For these 51 river basins daily streamflow series observed within the time span 1921–2000 are available from the National Hydrographic Service of Italy (SIMN). The streamflow regimes of the study area can be roughly classified into two large groups: (a) the maritime regime, with the maximum monthly streamflow during winter and the minimum during summer and (b) the Apenninic regime, with two maxima, a lower maximum during spring and a higher one during autumn.

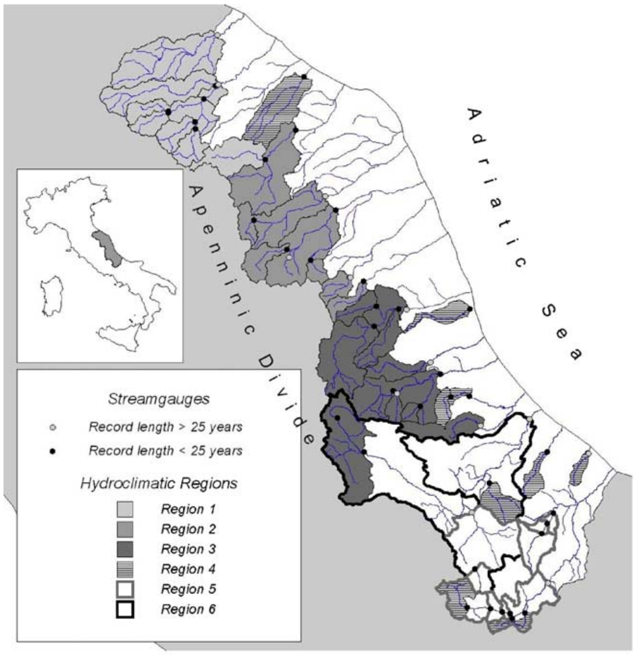

Fig. 1. Study region; 51 gauged river basins; 6 hydroclimatic regions.

The record length at the gauges varies from a minimum of five years to a maximum of 67 years with a mean value of 24 years. Several geomorphological and climatic characteristics were determined for the 51 basins, such as the basin area A, an estimate of the permeable portion of the basin area P, maximum, H_{max}, mean, H_{mean}, and minimum, H_{min}, elevations in metres above the sea level (a.s.l.), $\Delta H = H_{mean} - H_{min}$, and the main channel length L.

An extensive collection of hydrological information allowed us to characterise the 51 river basins from a climatic and litho-pedologic point of view. The climatic information was retrieved from an evenly spaced network of 88 thermometric sensors and 337 raingauges. The monthly series of areal temperature and rainfall depth were evaluated for each river basin through the Thiessen polygons method by referring to the thermo-pluviometric data collected in the same time span of the streamflow observations. These measures were then utilised to derive the 51 values of the mean annual temperature, MAT, mean annual precipitation, MAP, mean annual potential evapotranspiration, MAPET, and mean annual net precipitation, MANP = MAP − MAPET.

The significant differences existing between the minimum, average and maximum values of many geomorphoclimatic indexes reported in Table 1 (see e.g., A, P, L, MANP) are representative of the high hydrological complexity of the study region.

6. Implementation of the regional models and assessment of their reliability for ungauged sites

Our study applied several regional procedures, included the procedures proposed in the studies listed at the beginning of Section 4. Nevertheless, the remainder of the paper will present in detail only the three most successful applications of each regional approach (i.e., statistical, parametric and graphical).

6.1. Statistical approach

The implementation of the approach proposed by Fennessey and Vogel [9] provided the best representation of the empirical FDC's. The optimal parameters $\hat{\mu}$ and $\hat{\sigma}$ were estimated minimising within the range $0.30 \leqslant p \leqslant 0.99$ the sum of the squared residuals between the exceedance probabilities of the observed daily streamflows and the probabilities estimated from (2). The goodness of the fit between the 51 empirical and estimated FDC's was quantified in terms of the Nash-Sutcliffe efficiency criterion [18], E, computed with respect to 691 durations, $D = 0.300, 0.301, \ldots, 0.990$. The 51 resulting E values were all greater than 0.94, and exceeded 0.98 in 47 cases.

The regionalisation of the approach required the development of two regional predictive models for $\hat{\mu}$ and $\hat{\sigma}$. To this aim, the natural logarithms of all geomorphoclimatic indexes for the 51 sites were regressed against the corresponding $\hat{\mu}$ and $\hat{\sigma}$ values trough a multivariate stepwise regression analysis [30]. The analysis considered models of the form

$$\hat{\theta} = A_0 + A_1 \ln(X_1) + A_2 \ln(X_2) + \cdots + A_n \ln(X_n) + \vartheta \tag{11}$$

where $\hat{\theta}$ is either $\hat{\mu}$ or $\hat{\sigma}$, X_i, for $i = 1, 2, \ldots, n$, are the explanatory variables of the model (i.e., a suitable set of geomorphic and climatic indexes), A_i, for $i = 0, 1, \ldots, n$, are parameters and ϑ is the residual of the model. The optimal subset of explanatory variables

Table 1
Minimum, average and maximum values of the geomorphological and climatic indexes for 51 unregulated basins in the study region

Index	A (km^2)	P (%)	H_{max} (m a.s.l.)	H_{med} (m a.s.l.)	H_{min} (m a.s.l.)	ΔH (m)	L (km)	MAT (°C)	MAP (mm)	MAPET (mm)	MANP (mm)
Minimum	31.6	0.1	279.5	178.0	3.0	158.3	9.6	8.3	824.2	581.7	13.9
Average	357.7	48.3	2077.9	948.6	361.6	587.0	36.8	11.6	1095.3	692.1	403.0
Maximum	3082.0	99.0	2914.0	1950.0	1103.1	1543.2	159.9	15.3	1505.4	826.0	923.7

and the estimates of A_i, with $i = 0, 1, \ldots, n$, for both regional models were identified through multivariate stepwise regression analyses, implemented using an ordinary least-squares (OLS) algorithm. The stepwise procedure adopted in our study begins by considering a simple model that estimates the dependent variable, e.g. $\hat{\theta}$ of (11), as a constant value. At each step the procedure adds a further explanatory variable on to the model, choosing the variable that minimises Mallow's C_p statistics (see e.g., [30]). If we suppose that the addition leads to a model with p explanatory variables, the procedure then tests the performance of all the $p-1$ models including $p-1$ explanatory variables that can be obtained from the p-variable model by dropping one variable at a time. In case none of the simpler models performs better than the p-variable model, the procedure will search for the best multivariate model with $p+1$ explanatory variables. The stepwise procedure continues in the same way and comes to an end when no further reductions in Mallow's C_p can be obtained.

The identified models read,

$$\hat{\mu} = -11.255 + 1.117\ln(A) + 0.808\ln(\Delta H) + 0.108\ln(\text{MANP}) + \vartheta' \quad (E = 0.968) \tag{12}$$

and

$$\hat{\sigma} = 3.155 + -0.238\ln(P) + -0.419\ln(\Delta H) + 0.209\ln(\text{MANP}) + \vartheta'' \quad (E = 0.528) \tag{13}$$

where $\hat{\mu}$ and $\hat{\sigma}$ are expressed in $\ln(\text{m}^3/\text{s})$, A in km^2, ΔH in m, MANP in mm, and P in %.

The models (12) and (13) were then cross-validated through a jack-knife procedure considering in turn each of the 51 stations. The jack-knifed E resulted equal to 0.967 and 0.403 for the regression models for $\hat{\mu}$ and $\hat{\sigma}$, respectively. The scatter plot of Fig. 2 reports the optimal values $\hat{\mu}$ (panel a) and $\hat{\sigma}$ (panel b) against the estimates obtained from the regression models, applied with the jack-knife procedure or without, i.e. models (12) and (13). The E values and the scatter plots of Fig. 2 indicate that the application of the jack-knife procedure does not significantly influence the performances of the models, implying a strong robustness of the models themselves [3].

Using the jack-knifed estimates of $\hat{\mu}$ and $\hat{\sigma}$, the 51 jack-knifed FDC's for the statistical model were constructed. The jack-knifed FDC's were then compared with the empirical FDC's using the indexes of reliability described at Section 4.2, where $N_D = 691$ (i.e., $D = 0.300, 0.301, \ldots, 0.990$). Table 2 reports the performance indexes, whereas Fig. 3 reports the relative error bands for the jack-knifed FDC's, providing a comprehensive description of the model overall reliability for ungauged river basins located within the study area.

6.2. Parametric approach

All analytical expressions with two or more parameters reproduced the empirical 51 FDC's extremely well. By contrast, the regionalisation and the application of the jack-knife validation to most of the parametric approaches resulted in controversial outcomes. For several analytical expression we were unable to identify by means of multiple regression analyses regional models for all parameters, especially for the expressions in which the physical meaning of the parameters is unclear (see e.g., Eq. (4)).

The approach by Franchini and Suppo [11], unlike other parametric approaches, regionalises streamflow quantiles instead of parameters (see Section 3.2).

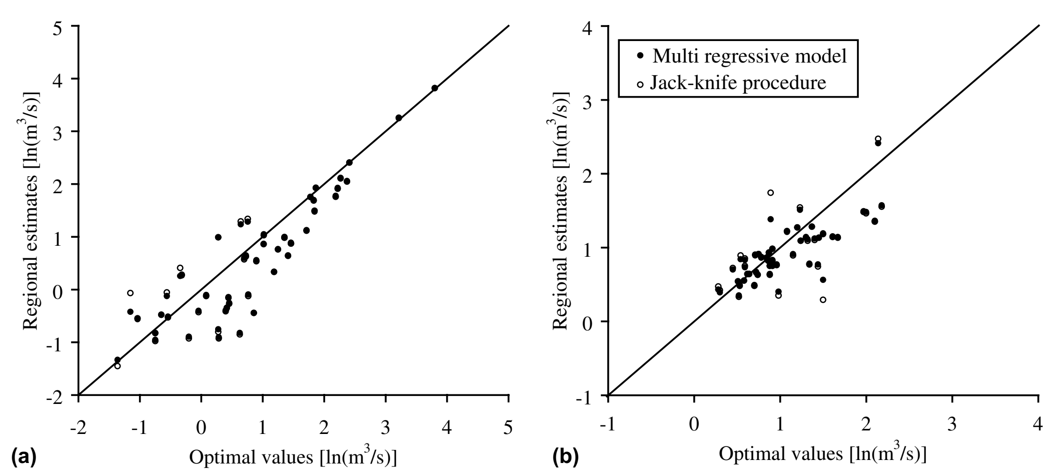

Fig. 2. Statistical approach; estimates of the optimal parameters $\hat{\mu}$ (panel a) and $\hat{\sigma}$ (panel b) obtained by applying the regional models and the jack-knife procedure.

Table 2
Jack-knife cross-validation: indexes of performances

Regional approach	$\bar{\varepsilon}$	σ_ε	P_1 (%)	P_2 (%)	P_3 (%)
Statistical	−0.104	0.175	29.4	9.8	60.8
Parametric	0.109	0.314	31.4	9.8	58.8
Graphical	−0.134	0.141	21.6	21.6	56.9

Probably due to this feature, the approach outperformed all other parametric approaches, showing the best performance indexes after the jack-knife cross-validation, and revealed to be particularly suitable for the study area.

The first implementations of the approach over the duration range [0.30–0.99] showed that using four pairs $[D_i, Q(D_i)]$, with $i \in [1,2,3,4]$, instead of three, and estimating the parameters of (5) by an OLS algorithm based on these four pairs, improves the reproduction of the empirical FDC's. The best results were obtained by referring to the four durations $D \in \{0.3, 0.7, 0.9, 0.95\}$ and by regionalising the corresponding empirical flood quantiles $Q(D)$ (i.e., Q_{30}, Q_{70}, Q_{90} and Q_{95}) with predictive models of the form

$$Q(D) = A_0 X_1^{A_1} X_2^{A_2} \cdots X_n^{A_n} + \vartheta, \tag{14}$$

where X_i, for $i = 1, 2, \ldots, n$, are the explanatory variables and ϑ is the residual of the model. The OLS stepwise regression algorithm identified the following regression models:

$$Q_{30} = 1.154 \times 10^{-3} A^{0.795} L^{0.317} \mathrm{MAP}^{1.234} \Delta H^{0.426} + \eta' \quad (E = 0.965) \tag{15}$$

$$Q_{70} = 1.764 \times 10^{-6} A^{1.267} L^{-0.016} \mathrm{MAP}^{0.620} \Delta H^{1.109} + \eta'' \quad (E = 0.979) \tag{16}$$

$$Q_{90} = 2.053 \times 10^{-7} A^{1.326} L^{-0.160} \mathrm{MAP}^{-0.045} \Delta H^{1.278} + \eta''' \quad (E = 0.973) \tag{17}$$

$$Q_{95} = 1.315 \times 10^{-7} A^{1.427} L^{-0.194} \mathrm{MAP}^{-0.159} \Delta H^{1.273} + \eta'''' \quad (E = 0.974) \tag{18}$$

where $Q(D)$ is expressed in m^3/s. The jack-knife cross-validation of models from (15)–(18) produced E values equal to 0.952, 0.967, 0.932 and 0.900, respectively. The scatter plots of Fig. 4 illustrate on the natural scale, and for $Q(D) \in [0.1, 10.0]$ on the log scale, the performance of the best and the worst performing models, that is models (16) and (18), respectively, along with the results of their jack-knife cross-validation. Fig. 4 indicates that the application of the jack-knife procedure does not significantly influence the performances of the models and confirms the robustness of the models.

The jack-knifed estimates of the four reference daily streamflows, $\widehat{Q}_{30}$, $\widehat{Q}_{70}$, $\widehat{Q}_{90}$ and $\widehat{Q}_{95}$, were utilised to construct the 51 jack-knifed FDC's. The jack-knifed parameters of (5), $\hat{a}$, $\hat{b}$, and $\hat{c}$, were estimated by an OLS algorithm using the four pairs $(0.3, \widehat{Q}_{30})$, $(0.7, \widehat{Q}_{70})$, $(0.9, \widehat{Q}_{90})$, $(0.95, \widehat{Q}_{95})$ and the resulting FDC's were then compared with the corresponding empirical FDC's, producing the statistical indexes set out in Table 2 and the relative errors depicted in Fig. 3.

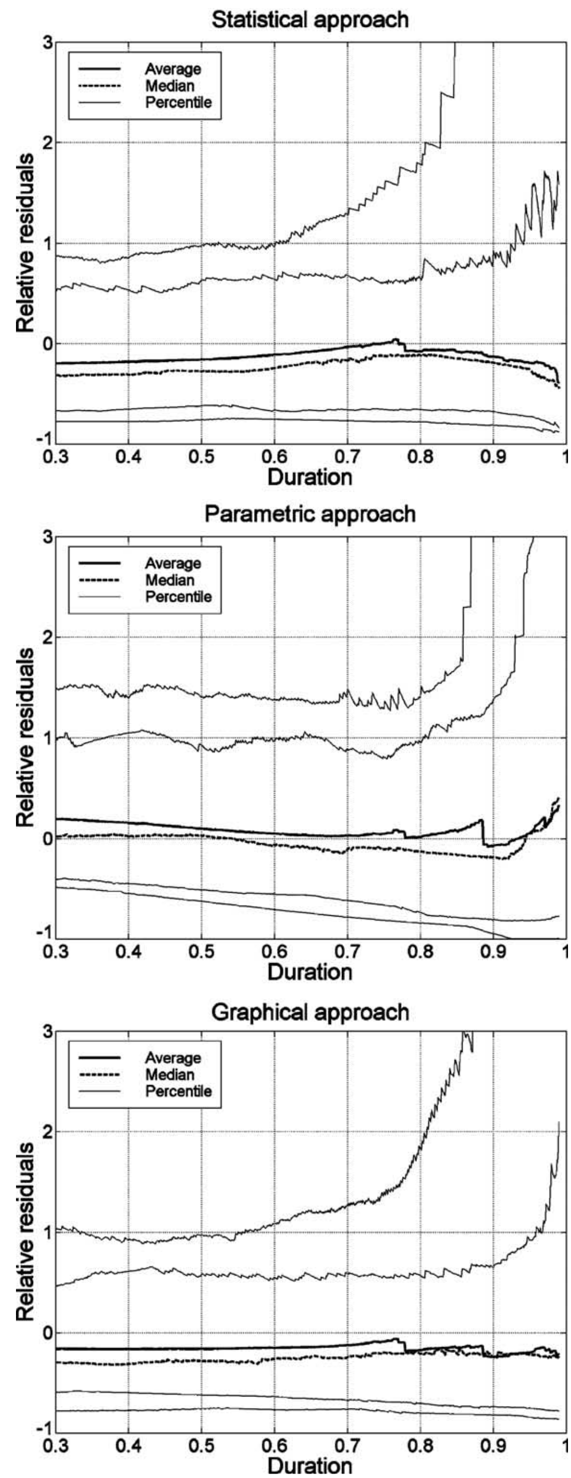

Fig. 3. Distribution of the relative errors for the examined regional models: mean, median and bands containing 75% and 90% of the relative errors.

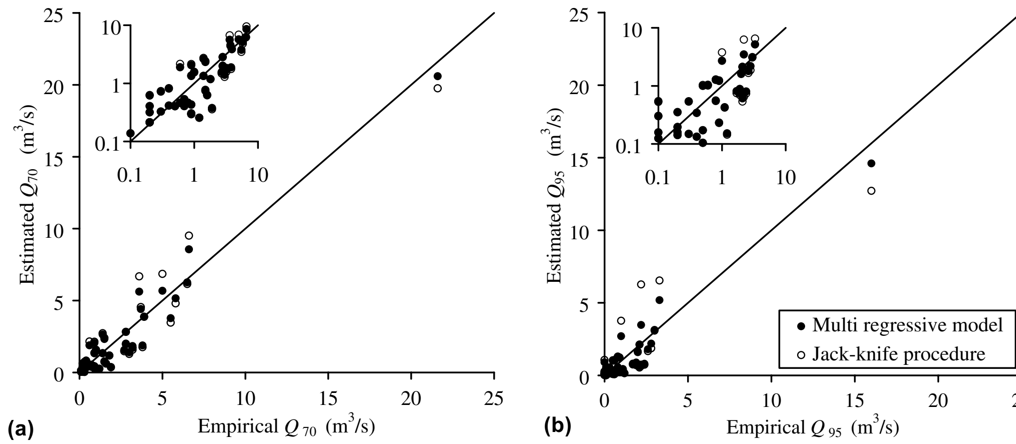

Fig. 4. Parametric approach; estimates of $Q(D)$ obtained by applying the regional models and the jack-knife procedure: (panel a) Q_{70} (best performing model); (panel b) Q_{95} (worst performing model).

6.3. Graphical approach

Several adaptations of the original procedure proposed by Smakhtin et al. [25] were applied in the study. The first two applications considered the entire study region to be homogeneous and referred to two different index-flows for the standardisation of the empirical FDC's: Q_{70} and the long-term mean of daily streamflows, μ_Q. A further multivariate regression model with expression analogous to (14) was then identified for the estimation of μ_Q and the construction of FDC's at ungauged river basins.

The two implementations of the graphical approach were cross-validated though a jack-knife procedure, consisting of the following steps: (a) jack-knife cross-validation of the multivariate regression model for the estimation of the index-flow, performed similarly to the validation of the models (15) through (18); (b) construction of 51 jack-knifed regional dimensionless FDC's, one for each site, by neglecting in turn one site and averaging all remaining standardised empirical FDC's; (c) estimation of the jack-knifed FDC's as the product of the 51 jack-knifed estimates of the index-flow (i.e., Q_{70} or μ_Q) obtained at step (a) and the corresponding jack-knifed regional FDC's obtained at step (b). The results showed that (i) using μ_Q to standardise the empirical FDC's leads to more accurate results than using Q_{70}, and (ii) assuming the entire region to be homogeneous is not a viable solution due to the differences existing in the streamflow regimes of the 51 river basins.

An accurate analysis of the streamflow regimes existing in the study region resulted in the identification of six hydroclimatic regions (see Fig. 1): Region 1 collects 9 small and medium-sized northern mountainous river basins, with relatively high percentages of impervious area; Region 2 includes 9 small and medium-sized northern catchments that are more pervious than those of Region 1; Region 3 consists of 12 mountainous and hilly river basins that are highly permeable and span a wide range of sizes; Region 4 includes 7 rather impervious coastal catchments that are all characterised by low average altitudes and 3 fairly small mountainous catchments located in the southern portion of the study area;

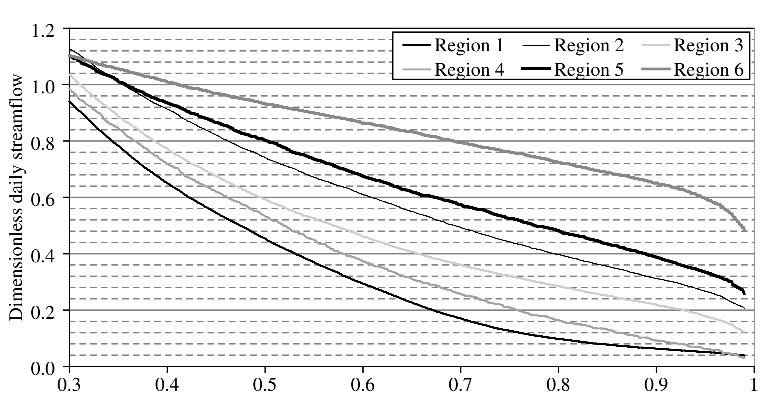

Fig. 5. Regional dimensionless FDC's for the 6 hydroclimatic regions.

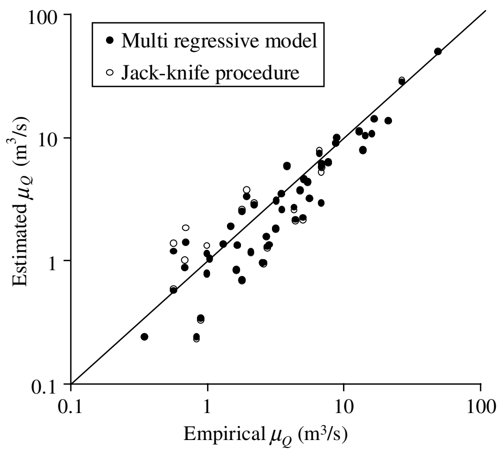

Fig. 6. Graphical approach; estimates of μ_Q obtained by applying the regional model and the jack-knife procedure.

Region 5 consists of 9 highly permeable, medium-sized mountainous and hilly southern basins; Region 6 includes the two largest river basins considered in the study. The six hydroclimatic regions are depicted in Fig. 1, while the dimensionless regional FDC's are reported in Fig. 5.

The model of μ_Q, identified by a stepwise regression analysis, reads,

$$\mu_Q = 1.296 \times 10^{-4} A^{0.726} \Delta H^{0.455} L^{0.519} \mathrm{MANP}^{0.251} + \theta \quad (E = 0.936) \qquad (19)$$

where μ_Q is expressed in m^3/s. The model refers to the whole study region; it was observed that the subdivision of the study area into sub-zones always resulted in the identification of less robust regression models (i.e., more sensitive to the jack-knife cross-validation). The jack-knife cross-validation returned $E = 0.931$, showing a rather high robustness of the model (Fig. 6).

The implementation of the graphical approach using a subdivision into six hydroclimatic regions was cross-validated using the scheme presented at the beginning of this section, which produced the statistical indexes set out by Table 2 and Fig. 3. It is worth noting that, during the validation, we assigned each basin to the right hydroclimatic region. In fact, the selection of the appropriate hydroclimatic region for an ungauged basin can be a problematic task, especially for the basins located outside the hydroclimatic regions of Fig. 1 or near the border between two or more regions.

7. Empirical flow-duration curves for short samples

This section presents a series of resampling experiments performed to assess the sensitivity of empirical FDC's to the sample length. The analysis considered 14 river basins with at least 25 years of daily streamflows (Fig. 1), and consisted of the following steps: (a) from

Table 3
Resampling experiment: indexes of performances

Sub-sample length	$\bar{\varepsilon}$	σ_ε	P_1 (%)	P_2 (%)	P_3 (%)
1 year	0.127	0.327	53.9	21.7	24.4
2 years	0.095	0.258	66.3	16.4	17.3
5 years	0.070	0.186	78.2	9.2	12.6

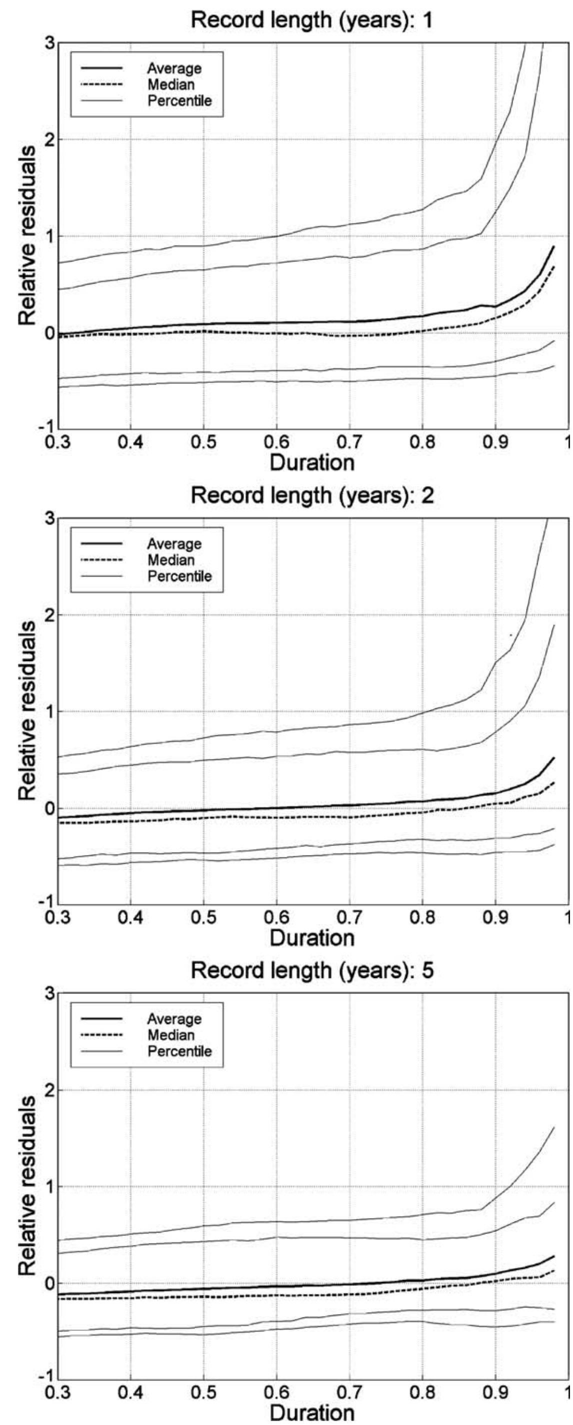

Fig. 7. Distribution of the relative errors for the resampling experiment: mean, median and bands containing 75% and 90% of the relative errors.

the L years of daily flows of the historical sample, we extracted the $m = L - l + 1$ consecutive sub-samples with length equal to l years; (b) using the procedure described in Section 2, the empirical FDC's of each sub-sample were constructed and compared with the empirical FDC of the entire period of record (long-term FDC). The comparison of step (b) was performed for 70 arbitrarily selected durations: $D = 0.30, 0.31, \ldots, 0.99$.

For each streamgauge $s = 1, 2, \ldots, 14$, the comparison produced m values of $\bar{\varepsilon}_s$ and $\sigma_{\varepsilon,s}$, computed as in (8) and (9). The average of the m values obtained for each statistical index was regarded as representative for station s and length l, and the average of the 14 mean values was considered to be representative for the entire region. These indexes are reported in Table 3 for l equal to 1, 2 and 5 years. Analogously, for each streamgauge $s = 1, 2, \ldots, 14$ and each sub-sample length $l = 1$, 2 and 5, the E index defined in (10) was computed. The average of the m values of E was regarded as representative for station s and used to compute the percentages of the 14 cases for which $E > 0.75$ (P_1), $0.75 \geqslant E > 0.50$ (P_2) and $E \leqslant 0.50$ (P_3), reported in Table 3.

Fig. 7 shows for $l = 1$, 2 and 5 the average over the m cases and 14 streamgauges of the mean and median of the distribution of relative errors, along with the 75% and 90% bands as functions of the duration, and can be compared with the diagrams of Fig. 3.

Fig. 7 shows a generalised underestimation of the FDC's for short durations and an extensive overestimation of the FDC's for long durations. This was expected, as long-term FDC's are affected by the observation of abnormally wet or dry periods during the period-of-record [28]. A sub-sample does not necessarily include all extreme floods or low-flows that are present in the entire period-of-record, and therefore the sub-sample FDC tends to underestimate the corresponding long-term FDC for short durations and to overestimate it for long durations.

8. Discussion of results

Before discussing the results of the study, it is worth commenting on the multivariate regression models identified herein, that is Eqs. (12), (13) and (15) through (19). Only a few of the many parameters available to describe the basin morphology and climate were found to be useful as explicative variables of the regional models (i.e., A, P, ΔH, L, MAP, MANP, see Table 1). This indicates that, due to the close statistical correlation between these parameters it is sufficient to consider a very small number of them in order to represent the geomorphoclimatic effects on the streamflow regime (e.g., [3]).

Table 4 reports the coefficients of correlation between the six catchment attributes utilised by the final regional regression models. Table 4 shows a high degree of correlation between A and L, and between MAP and MANP. While MAP and MANP are never used simultaneously, A and L are both present in the models (15) through (19). The concurrent presence of A and L is a direct product of the stepwise regression analysis and it was to be expected. Even though A and L are highly correlated, A can only represent the size of the singular catchment, whereas a combination of A and L is capable of describing the catchment's size and shape, both impacting the streamflow regime and thereby playing a significant role on $Q(D)$, i.e. models (15) through (18), and μ_Q, i.e. model (19).

Table 4
Correlation coefficients for the geomorphoclimatic indexes utilised in the regional regression models

	A	P	ΔH	L	MAP	MANP
A	–	−0.016	0.219	0.917	−0.262	−0.245
P	−0.016	–	0.064	−0.206	0.316	0.337
ΔH	0.219	0.064	–	0.117	−0.149	−0.035
L	0.917	−0.206	0.117	–	−0.270	−0.280
MAP	−0.262	0.316	−0.149	−0.270	–	0.966
MANP	−0.245	0.337	−0.035	−0.280	0.966	–

Future analyses focusing on different geographical regions and climatic conditions will provide useful indications about the general validity of regional models identified in this study. Concerning the study region, the results of the cross-validation show that for ungauged sites the reliabilities of the three best-performing regional models are comparable to one another. The three distributions of relative errors of Fig. 3 are very similar, and also Table 2 reports similar indexes of reliability. The parametric approach shows the highest dispersion of the relative errors around zero, more evidently for $D > 0.75$ (see Fig. 3), and consequently the approach presents the highest average standard deviation of relative errors, σ_ε (see Table 2). The values of $\bar{\varepsilon}$ reported in Table 2 and the average curves in Fig. 3 show that the statistical and graphical approaches are negatively biased, whereas the parametric approach is positively biased. The jack-knifed FDC's of all three models are poor fits to the corresponding empirical curves in roughly 60% of the cases (i.e., P_3 in Table 2), that is 31 sites out of 51, and the smallest P_1 is presented by the graphical procedure. Nevertheless, the reliability of the three different models appears to be substantially similar to one another, even though they have different theoretical backgrounds and were implemented using different procedures.

Concerning the second aim of the research, interesting indications emerge from the resampling experiments presented in Section 7. The comparison of the diagrams reported in Figs. 3 and 7 and the indexes of Tables 2 and 3 show that five years of observed streamflows are generally sufficient to obtain better estimates of the long-term FDC than the estimates produced by the best performing regional model. This result highlights the remarkable value associated with observed streamflows, even for exceptionally short series.

In summary, the estimation of the FDC for ungauged basins in the study region should be performed by applying all three approaches presented and selecting the estimated FDC only after a scrupulous analysis of the suitability of each regional model to the particular subregion where the ungauged site is located. To this aim, the reliability of the regional FDC's estimated with the three models for gauged river-basins located in the subregion should be taken into account. Nevertheless, the study indicates that it is not advisable to rely completely on any of the three models, which should be used to provide a first order approximation of the long-term FDC. The practical utilisation of the estimated FDC for designing purposes requires a subsequent refinement that should be based upon an ad hoc measurement campaign.

9. Conclusions

The main aim of this study was to analyse the reliability of several regional procedures for the estimation of flow-duration curves (FDC's) at ungauged sites. Achieving this aim is fundamental as (a) the estimation of FDC's at ungauged sites is a crucial and recurrent hydrological task for addressing problems related to hydropower generation, river and reservoir sedimentation water-use assessment, water allocation and habitat suitability, (b) the literature on the regionalisation of FDC's is surprisingly sparse and (c) the regional models themselves lack validation or are validated for relatively small geographical regions, (d) for an effective use of the streamflow information available at gauged river basins with limited record lengths, it is fundamental to know whether empirical FDC's based on small samples can represent a better estimate of the long-term FDC's than the regional FDC's.

Several regional models for the estimation of FDC's at ungauged sites within the duration range $D \in [0.30, 0.99]$, based on three different theoretical approaches, were applied to a large geographical region of eastern-central Italy including 51 unregulated river basins. All regional models were cross-validated by means of a jack-knife resampling procedure. The results of the cross-validation were then compared with the results of a resampling experiment designed to asses the reliability of empirical FDC's constructed from small samples. The two main results of our study may be summarised as follows:

1. The cross-validation shows that the three regional models (i.e., one model for each theoretical approach) that provide the best performances over the study region are characterised by similar degrees of reliability and robustness.
2. The analysis shows for the study region that five years of observed streamflows are sufficient to obtain rather consistent estimates of the long-term FDC, and these estimates are generally more reliable than the estimates produced by the regional models.

The first outcome, which seems to contrast with the fact that the three regional models derive from three rather different theoretical approaches and were implemented independently to one another, was indeed to be expected as the regional models have similar complexities and flexibilities; the reliability of the models themselves is therefore mainly controlled by the hydrological complexity of the study region. As a consequence, we suggest to estimate the FDC for ungauged basins in the study region by applying all three models. If the estimates differ significantly from one another, we recommend to perform a careful investigation of the suitability of each regional model to the particular sub-region were the ungauged catchment is located, before selecting the estimated FDC.

The second outcome indicates that the implementation over large geographical areas of simple regional models, like the models considered herein, can be effectively used for a preliminary identification of ungauged river basins with a suitable water supply potential. Nevertheless, we recommend to validate and support the regional estimates of long-term FDC's through a smaller-scale analysis and, possibly, a measurement campaign, even for a limited period of time.

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