The Saaty's method dealt with consistency of the pairwise comparison matrix. A consistent matrix mean e.g. if the decision maker says a criterion $\boldsymbol{x}$ is equal important to another criterion $\boldsymbol{y}$ (so the comparison matrix will contain value of $a_{xy} = 1 = a_{yx}$), and the criterion $\boldsymbol{y}$ is absolutely more important as an criterion $\boldsymbol{w}$ ($a_{yw} = 9$; $a_{wy} = 1/9$); then the criterion $\boldsymbol{x}$ should also be absolutely more important than the criterion $\boldsymbol{w}$ ($a_{xw} = 9$; $a_{wx} = 1/9$). Unfortunately, the decision maker is often not able to express consistent preferences in case of several criteria. Then, the Saaty's method measure the inconsistency of the pairwise comparison matrix and set a consistency threshold which should not be exceeded.

In ideal case the comparison matrix ($\boldsymbol{A}$) is fully consistent, the rank($\boldsymbol{A}$) $= 1$ and $\lambda = \boldsymbol{n}$ ($\boldsymbol{n}$ = number of criteria). In this case, the following equation is valid:

$\boldsymbol{A} \times \boldsymbol{x} = \boldsymbol{n} \times \boldsymbol{x}$ (where $\boldsymbol{x}$ is the *eigenvector* of $\boldsymbol{A}$)

The vector $\boldsymbol{x}$ represent the weights we are looking for.

In the non-consistent case (which is more common) the comparison matrix $\boldsymbol{A}$ may be considered as a perturbation of the previous consistent case. When the entries a_{ij} changes only slightly, then the *eigenvalues* change in a similar fashion. Moreover, the maximum *eigenvalue* (λ_{max}) is closely **grater** to $\boldsymbol{n}$ while the remaining (possible) *eigenvalues* are close to zero. Thus is order to find weights we are looking for the *eigenvector* which corresponds to the maximum *eigenvalue* (λ_{max}).

In order to obtain weights from calculated eigenvector the values have to be normalised by formula 1. (The weights have to sum up to 1.)

$$w_j = \frac{\tilde{w}_j}{\sum_{i=1}^{n} \tilde{w}_i} \qquad 1.$$

The consistency index (CI) is calculated as following

$$CI = \frac{\lambda_{\max} - n}{n-1} \qquad 2.$$

Then, the consistence ratio (CR) is calculated as the ratio of consistency index and random consistency index (RI). The RI is the random index representing the consistency of a randomly generated pairwise comparison matrix It is derived as average random consistency index (Table 1) calculated from a sample of 500 of randomly generated matrices based on the AHP scale (Table 2)

$$CR(A) = \frac{CI(A)}{RI(n)} \qquad 3.$$

If **CR(A) ≤ 0.1**, the pairwise comparison matrix is considered to ***be consistent enough***. In the case CR(A) ≥ 0.1, the comparison matrix should be improved.. The value of RI depends on the number criteria being compared.

n	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
RCI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45

Table 1: Random consistency indices for different number of criteria (n).

1 – ***same*** importance
2 – slightly more important
3 – ***weekly*** more important
4 – weekly to moderately more important
5 – ***moderately*** more important
6 – moderately to strongly more important
7 – ***strongly*** more important
8 – greatly more important
9 – ***absolutely*** more important

Table 2: Scale of relative importance for pairwise comparison.