Talk:Analytic hierarchy process/Example Leader

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This is an example showing the use of the AHP in a practical decision situation.

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This case study describes the use of the AHP in choosing a leader for a company whose founder is about to retire. There are several competing candidates and several competing criteria for choosing the most suitable one. By using the AHP, the Board of Directors is able to choose the best candidate in a rational, transparent way that can be examined and understood by all concerned.

The diagram below shows the AHP hierarchy at the end of the decision making process. The goal is to choose the most suitable leader based on four specific criteria. Dick is the preferred alternative, with a priority of .493. He is preferred about a third more strongly than Tom, whose priority is .358, and about three times more strongly than Harry, whose priority is only .149. Experience is the most important criterion with respect to reaching the goal, followed by Charisma, Education, and Age. These factors are weighted .547, .270, .127, and .056, respectively.

The balance of this article describes the derivation of these priorities.

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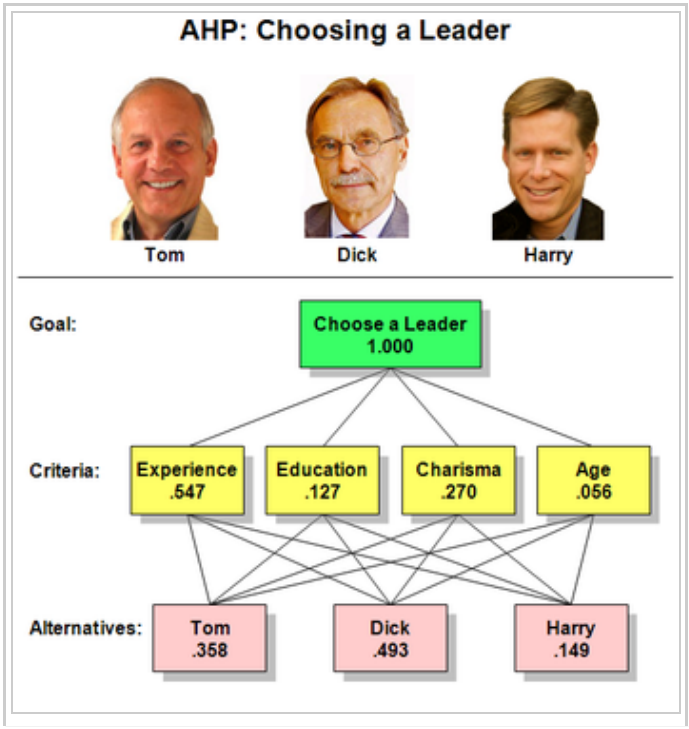


Decision scenario

The company, founded in 1960, makes specialized industrial equipment. Its future success will depend on maintaining the

strength of its older product lines and on generating a constant flow of new ones. The company's founder is retiring soon, and a consulting firm has developed a detailed plan for continuing its success in his absence. The plan will take five years to implement, and will replace the founder's highly subjective "seat of the pants" style with a more carefully thought out way of doing business.

The goal of this decision is to select the most suitable leader from a field of three candidates. The factors to be considered are Experience, Education, Charisma, and Age. According to the judgments of the decision makers, Dick is the strongest candidate, followed by Tom, then Harry. Their decision process is described in depth below.

The Board of Directors needs to choose someone to lead the company through the change and upheaval that implementing the consultant's plan will involve. In doing this work, the new leader will be required to make many unpopular decisions and take many unpopular actions. He or she will be expected to “clear the air” by stepping aside after the plan is fully implemented.

Six months ago, the Board said:

> After much thought and discussion, we have identified four criteria to be used in choosing the person to guide us through the upcoming period of change: Experience, Education, Charisma and Age. Experience is important because the job requires skills and knowledge that can only be developed through practical application. And though our beloved founder was a self-made man who didn’t finish high school, the times demand that our new leader have an appropriate university education. Since the new leader will have to keep us all motivated during a difficult period of change, we prefer someone with an active, charismatic leadership style. Finally, the new leader's Age is important because he or she will need to have an appropriate career path after stepping down five years from now. — *Board of Directors, Letter to Employees and Shareholders*

Last week, they said:

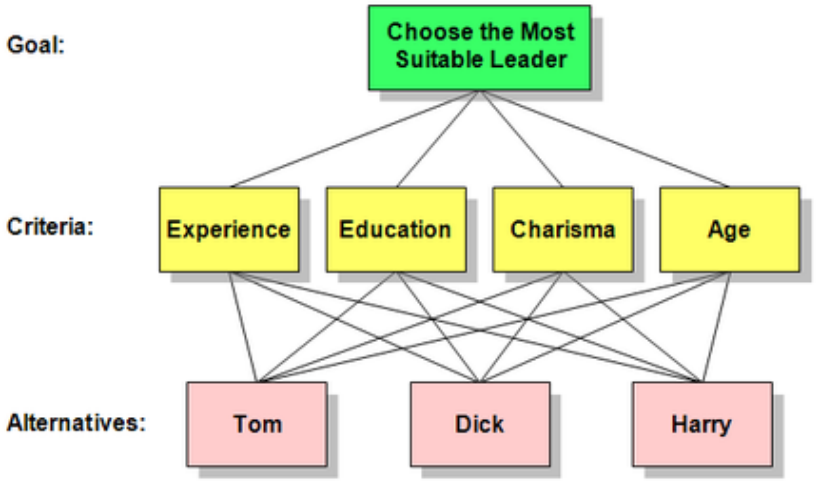
> After an extensive search, we have selected three candidates for this very challenging position. All are presently executives with the company. Choosing among them will be difficult, but we plan to announce our decision shortly. — *Board of Directors, Followup Letter to Employees and Shareholders*

The three candidates are Tom, Dick, and Harry. Summaries of their backgrounds are shown below:

	 Tom	 Dick	 Harry
Age	50 years	60 years	30 years
Experience	10 years with us, plus 16 in a different industry, all in sales and marketing, all with great success. Currently VP Sales, Marketing, and Customer Service.	30 years with us, 8 in another company in our industry. Has had key responsibilities in every department. Currently Executive VP.	5 years with us, 4 with a CPA firm. Completed the management training program in record time and with record performance. Currently VP Finance.
Education	BS, Marketing, Marietta College, 26 years ago. Online MBA, University of Phoenix, last year.	BA and MA, American History, Duke University, 39 years ago. Phi Beta Kappa.	BS, Economics, Princeton, 10 years ago. MBA, Pittsburgh, 5 years ago. Licensed CPA.
Leadership Qualities	An active, inspirational leader. Beloved by his subordinates and all those who have ever worked with him.	A bit reserved. Leads by example and strength of knowledge. Everyone in the company likes him and respects him.	Leads quietly from his office. Highly respected for his brilliance and knowledge of financial matters.

Decision hierarchy

The AHP hierarchy for this decision is shown below. [Note 1]



As the decision makers continue with the AHP, they will determine *priorities* for the candidates with respect to each of the decision criteria, and priorities for each of the criteria with respect to their importance in reaching the goal. [Note 2]

The priorities will then be combined throughout the hierarchy to give an overall priority for each candidate. The candidate with the highest priority will be the most suitable Alternative, and the ratios of the candidates' priorities will indicate their relative strengths with respect

to the Goal.

Pairwise comparisons

The priorities will be derived from a series of measurements: *pairwise comparisons* involving all the nodes.

The nodes at each level will be compared, two by two, with respect to their contribution to the nodes above them. The results of these comparisons will be entered into a matrix which is processed mathematically to derive the priorities for all the nodes on the level. [Note 3]

The comparisons can be made in any sequence, but in this example we will begin by comparing the Alternatives with respect to their strengths in meeting each of the Criteria. Then we'll compare the Criteria with respect to their importance to reaching the Goal. [Note 4]

Since there are three Alternatives (Tom, Dick, and Harry) and we need to compare each one to each of the others, we will make three pairwise comparisons with respect to each Criterion: Tom vs. Dick, Tom vs. Harry, and Dick vs. Harry. For each comparison, the Board will first judge which member of the pair is weaker with respect to the Criterion under consideration. Then they will assign a relative weight to the other candidate.

They will use the AHP Fundamental Scale in assigning the weights:

The Fundamental Scale for Pairwise Comparisons		
Intensity of Importance	**Definition**	**Explanation**
1	Equal importance	Two elements contribute equally to the objective
3	Moderate importance	Experience and judgment moderately favor one element over another
5	Strong importance	Experience and judgment strongly favor one element over another
7	Very strong importance	One element is favored very strongly over another; its dominance is demonstrated in practice
9	Extreme importance	The evidence favoring one element over another is of the highest possible order of affirmation
Intensities of 2, 4, 6, and 8 can be used to express intermediate values. Intensities of 1.1, 1.2, 1.3, etc. can be used for elements that are very close in importance.		

Alternatives vs. Criteria

Experience

Using their knowledge of the work the leaders will be required to do, the Board needs to evaluate the candidates' strengths with respect to Experience. Though they have good

information about each candidate's work history, there is no such thing as an objective scale for measuring "experience." Thanks to the AHP, the Board *will* be able to develop a scale, applying only to this one case, that measures the candidates' relative strengths with respect to experience.

Here is the Board's thinking about experience:

> The leader will implement a wide-ranging plan that involves major changes to a successful business. This work requires skills, knowledge, wisdom, and judgment that are usually present only in seasoned executives. Furthermore, the company is so complex and specialized that only direct experience inside it can equip a prospective leader for his job. Outside experience is also important, since it provides perspective and a view of the larger picture. — *Board of Directors, Internal Memorandum*

As a reminder, here is their summary of the candidates' experience:

Tom	Dick	Harry
10 years with us, plus 16 in a different industry. All in sales and marketing, all with great success. Currently VP Sales, Marketing, and Customer Service.	30 years with us, plus 8 in another part of the industry. Has had key responsibilities in every department. Currently Executive VP.	5 years with us, 4 with a CPA firm. Completed the management training program in record time and with record performance. Currently VP Finance.

The next step in the AHP is to compare pairs of candidates with respect to Experience. For each comparison, the Board decides which candidate is the weaker with respect to Experience, giving his experience a weight of 1. Then, using the AHP Fundamental Scale, they assign a weight to the experience of the other candidate.

Their comparisons are summarized below. (A summary in this form is not an essential part of the AHP. It is presented here only to help readers understand this example. The colors in the squares will help them see where each entry belongs in the AHP matrix):

Alternatives compared with respect to EXPERIENCE

Tom	1	Dick	4	Dick has a big advantage in length & variety of experience in our company, but is light outside it. Tom has more experience outside. Overall, his experience much narrower than Dick's. Dick's experience is more than moderately preferred to Tom's. Weight: 4
Tom	4	Harry	1	Overall, Harry is much less experienced than Tom. Both are rather specialized. Tom knows the company much better than Harry, and has heavy experience outside it. Tom's experience is somewhat more than moderately preferred to Harry's. Weight: 4
Dick	9	Harry	1	Harry is a relatively junior executive. Dick is an experienced senior. Harry has a slight edge in experience outside the company, but it is minor compared to Dick's grasp of our organization. Dick's experience is extremely preferred to Harry's. Weight: 9

The next step is to transfer the weights to a matrix, using a method unique to the AHP. For

each pairwise comparison, the number representing the greater weight is transferred to the box of the corresponding color; the reciprocal of that number is put into the box of the color corresponding to the smaller number: [Note 5]

Experience	Tom	Dick	Harry
Tom	1	1/4	4
Dick	4	1	9
Harry	1/4	1/9	1

By processing this matrix mathematically, the AHP derives priorities for the candidates with respect to Experience. The priorities are measurements of their relative strengths, derived from the judgments of the decision makers as entered into the matrix. Mathematically speaking, they are the values in the matrix's principal right eigenvector. These values can be calculated in many ways, including by hand, or with a spreadsheet program, or by using specialized AHP software. They are shown below to the right of the matrix, along with an Inconsistency Factor computed by the specialized AHP software that was used to process the data: [Note 6]

Experience	Tom	Dick	Harry	Priority
Tom	1	1/4	4	0.217
Dick	4	1	9	0.717
Harry	1/4	1/9	1	0.066
			Sum of Priorities	1.000
			Inconsistency	0.035

Education

Now the Board needs to evaluate the candidates with respect to Education. Here is their thinking on the subject:

> The leader needs to have a good education, preferably with a recent MBA or

> engineering degree. Our founder is a self-made man who didn't finish high school. He created a successful company that relies strongly on his personal insights and "seat-of-the pants" judgment. But when he retires and we move into a more complex future, we need to become more thoughtfully run. Having an educated leader is vitally important to this. Education is also important because our employees, particularly the technical staff, are hungry to have it in their leader. Though they love and respect "the Old Man," they have often been frustrated by his lack of appreciation for today's tools of business, engineering, and manufacturing. — *Board of Directors, Internal Memorandum*

Though a person's education can be measured by academic degrees, grades and honors, those measurements are not so useful in this non-academic situation. Through its system of pairwise comparisons, the AHP can develop accurate measurements that are perfectly suited to the decision at hand. Here is the Board's summary of the candidates' educational backgrounds:

Tom	Dick	Harry
BS, Marietta College, 26 years ago. MBA, University of Phoenix, last year.	BS, Duke University, 39 years ago. Teaches courses in industry best practices.	BS, Princeton, 10 years ago. MBA, Pittsburgh, 5 years ago. Licensed CPA.

As they did previously with Experience, the Board now compares pairs of candidates with respect to Education. For each comparison, the Board decides which candidate is the weaker with respect to Education, giving his education a weight of 1. Then, using the AHP Fundamental Scale, they assign a weight to the education of the other candidate. For the sake of our example, we again provide a table summarizing their deliberations:

Alternatives compared with repect to EDUCATION

Tom	3	Dick	1	Tom's degrees are more recent than Dick's. His recent MBA will help in implementing the plan. Dick is a renaissance man with a fine education, but it doesn't directly relate to the task at hand. Tom's education is moderately preferred to Dick's. Weight: 3
Tom	1	Harry	5	Harry's education is better for us than Tom's. Harry has better schools, a more recent BS, and CPA qualifications. Tom's MBA is newer, but Harry's is also recent. Harry's education is strongly preferred to Tom's. Weight: 5
Dick	1	Harry	7	Harry's education is much stronger than Dick's. BS & MBA vs. BS only. Both degrees are more recent than Dick's, and more germane to our 21st century needs. Harry's education is very strongly preferred to Dick's. Weight: 7

The Board transcribes its judgments into the AHP matrix shown below. Looking at Tom's row in the matrix, we can see that his education has three times the weight of Dick's, but only a fifth the weight of Harry's. Similar relationships can be seen in the other two rows, where Dick's education is strongly dominated by Tom's and Harry's, while Harry's education strongly dominates that of both the other men.

Education	Tom	Dick	Harry
Tom	1	3	1/5
Dick	1/3	1	1/7
Harry	5	7	1

Once again we use AHP software to derive priorities from the numbers in the AHP matrix:

Education	Tom	Dick	Harry	Priority
Tom	1	3	1/5	0.188
Dick	1/3	1	1/7	0.081
Harry	5	7	1	0.731
			Sum of Priorities	1.000
			Inconsistency	0.062

The priorities tell us that, according to the judgment of the decision makers, Harry's education is by far the strongest of the three. Generally speaking, it dominates Tom's by a factor of almost 4, (0.731÷0.188=3.95) and Dick's by a factor of 9 (0.731÷0.081=9.02).

Charisma

The Board's next step is to pairwise compare the Alternatives (candidates) with respect to Charisma. Here's what they have to say about it:

> As implementation of the plan proceeds, the leader will need to overcome people's natural resistance to change. We think he will often need to wield influence on the basis of his personal charm and appeal, rather than on talk, logic, or authority alone. We have called this charm and appeal "charisma". It will be an important factor in the leader's ability to implement the plan. — *Board of Directors, Internal Memorandum*

Here is their summary of the candidates' leadership qualities:

Tom	Dick	Harry
An inspirational leader. Beloved by his subordinates and everyone who has ever worked with him.	A bit reserved. Leads by example and strength of knowledge. People through-out the organization like and respect him.	Leads quietly from his office. Highly respected for his brilliance and knowledge of financial matters.

And here are their judgments about those qualities as they relate to Charisma:

Alternatives compared with respect to CHARISMA

Tom	5	Dick	1	Tom is very strong in the quality we are looking for. Dick isn't especially weak in it, but his laid back nature puts him in second place to Tom. Tom's charisma is strongly preferred to Dick's. Weight: 5
Tom	9	Harry	1	Tom is a very strong inspirational leader. Harry relies on thoughtful presentation of issues, rather than active, passionate leadership of people. Tom's charisma is extremely preferred to Harry's. Weight: 9
Dick	4	Harry	1	Dick's long tenure has given him confidence and the respect of our workforce. He is laid back, but people will follow him just because he is Dick. Harry doesn't yet have that ability. Dick's charisma is somewhat strongly preferred to Harry's. Weight: 4.

Putting their judgments into the matrix and processing it with the AHP software, they get:

Charisma	Tom	Dick	Harry	Priority
Tom	1	5	9	0.743
Dick	1/5	1	4	0.194
Harry	1/9	1/4	1	0.063
			Sum of Priorities	1.000
			Inconsistency	0.069

Note that even though "charisma" is a highly subjective concept with no imaginable measurement scale, the AHP has allowed the board to measure its relative strength among these three candidates. Also note that with different candidates, or even with different board members, the measurements would likely be different as well. The AHP's measurements, though accurate and produced through well-tested mathematics, apply only to the specific case at hand.

Age

A person's age can be determined with a precision of days or even minutes. But such measurements aren't highly useful in making the decision at hand, since there is more to "Age" in this context than mere chronology.

Also, there are U.S. laws against employment discrimination by age. Anyone using age as a decision factor must be very explicit about their reasons and justifications. In this case, it wouldn't be good if an unsuccessful candidate decided to sue the company on the basis of age discrimination.

The Board has said this about Age:

> The leader will be expected to step down five years from now. For his benefit and ours, we need to consider how old he will be at that time. If he is at or near retirement age, he can just retire, possibly remaining with us in a consulting capacity. If he is younger, he will have more years to work, but there will likely be no place for him in our company. Based on his success as our leader, he might or might not be easily employable elsewhere. He could possibly join a competitor and take key people with him, in spite of any non-compete agreements we might negotiate.
>
> The leader's age also affects his ability to interact with our workforce. Many of the people involved with our important legacy products are over age 55. Most of those connected with our newer products are in their 20's and 30's. It is important that the leader is able to relate to both these age groups, and that they are able to accept his leadership. — *Board of Directors, Internal Memorandum*

Tom, Dick, and Harry are now 50, 60, and 30 years old respectively. Here's a summary of the Board's evaluation of them with respect to Age:

Alternatives compared with respect to AGE

Tom	1	Dick	3	When the leader steps down, Dick will be about 65 and ready to retire. Tom will be 55, and ten years away from retirement. He could retire early or find another senior position in the industry. Dick's age is moderately preferred to Tom's. Weight: 3
Tom	5	Harry	1	When he steps down at 55, Tom might retire early or find another senior position. Harry will be 35 with no place to go in the company, and we'll probably lose him. Also Harry doesn't relate very well to older workers Tom's age is strongly preferred. Weight: 5
Dick	9	Harry	1	Dick will be of retirement age when the leader steps down, and can gracefully retire. Harry will be 35 with no place to go in the company. Dick's age is extremely preferred. Weight: 9

Transfering their judgments to an AHP matrix and processing it with software yields this result for the Alternatives with respect to Age:

Age	Tom	Dick	Harry	Priority
Tom	1	1/3	5	0.265
Dick	3	1	9	0.672
Harry	1/5	1/9	1	0.063
			Sum of Priorities	1.000
			Inconsistency	0.028

Criteria vs. the Goal

Now that the decision makers have evaluated the Alternatives (candidates) with respect to their strength in meeting the Criteria, they need to evaluate the Criteria with respect to their importance in reaching the goal.

Once again they do this by a series of pairwise comparisons. As with the Alternatives, this part of the process requires much discussion and debate among the decision makers.[Note 7]

In this case, the Board agrees on these relative weights for the various pairs of Criteria:

Criteria compared with respect to reaching the Goal:

Experience	4	Education	1	Education matters primarily because the founder lacked it. Experience is important because implementing the plan is a complex managerial task that can't be done without it. Experience is somewhat strongly more important than education. Weight: 4
Experience	3	Charisma	1	Charisma is needed when facts, talk, logic, wisdom, etc. aren't enough. Experience gives the knowledge to implement the plan, and lends credibility to the leader's wants. Experience is moderately more important than Charisma. Weight: 3
Experience	7	Age	1	Age is important because the leader will step down, and because he must to relate to both young and old employees. Experience is a vitally important requirement of doing the job. Experience is very strongly more important than age. Weight: 7
Education	1	Charisma	3	Charisma helps the leader to lead. Education, too, by giving him respect. Education also gives us something we want in our leader. Charisma is moderately more important than education. Weight: 3
Education	3	Age	1	The importance of Age is described above. Education is important because we want to be run more thoughtfully than in the past, and because our employees want it in their leader. Education is moderately more important than Age. Weight: 3
Age	1	Charisma	5	The importance of Age is described above. Charisma is important because the leader will need to use it to lead; words and logic won't always be enough. Charisma is strongly more important than Age. Weight: 5

Note that pairwise comparison of four nodes requires six separate comparisons, while that

of three nodes requires only three.[Note 8]

These comparisons require a larger matrix, but it is processed in the same way as the smaller ones. Here is the matrix for the Criteria, along with the resulting priorities and Consistency Factor:

Criteria	Experience	Education	Charisma	Age	Priority
Experience	1	4	3	7	0.547
Education	1/4	1	1/3	3	0.127
Charisma	1/3	3	1	5	0.270
Age	1/7	1/3	1/5	1	0.056
				Sum of Priorities	1.000
				Inconsistency	0.044

Note that, in this decision, Experience, the highest ranked Criterion, is about twice as important in reaching the goal as the second-highest ranked Criterion, Charisma. Similarly, Charisma is about twice as important as Education, which in turn is more than twice as important as Age.

Synthesizing final priorities

Now that we know the priorities of the Criteria with respect to the Goal, and the priorities of the Alternatives with respect to the Criteria, we can calculate the priorities of the Alternatives with respect to the Goal. This is a straightforward matter of multiplying and adding, carried out over the whole of the hierarchy.[Note 9]

In our example, each Alternative's priority with respect to reaching the Goal is the sum of:

- His priority with respect to Experience, multiplied by Experience's priority with respect to the Goal, and
- His priority with respect to Education, multiplied by Education's priority with respect to the Goal, and
- His priority with respect to Charisma, multiplied by Charisma's priority with respect to the Goal, and
- His priority with respect to Age, multiplied by Age's priority with respect to the Goal

Here are the calculations for the Candidates with respect to the Criteria:

Criterion	Priority vs. Goal	Alternative	A		B		C
Experience	0.547	Tom	0.217	x	0.547	=	0.119
		Dick	0.717	x	0.547	=	0.392
		Harry	0.066	x	0.547	=	0.036
			1.000				0.547
Education	0.127	Tom	0.188	x	0.127	=	0.024
		Dick	0.081	x	0.127	=	0.010
		Harry	0.731	x	0.127	=	0.093
			1.000				0.127
Charisma	0.270	Tom	0.743	x	0.270	=	0.201
		Dick	0.194	x	0.270	=	0.052
		Harry	0.063	x	0.270	=	0.017
			1.000				0.270
Age	0.056	Tom	0.265	x	0.056	=	0.015
		Dick	0.672	x	0.056	=	0.038
		Harry	0.063	x	0.056	=	0.004
			1.000				0.056

Key: Column A shows the priority of this Alternative with respect to this Criterion. Column B shows the priority of this Criterion with respect to the Goal. Column C shows the product of the two, which is the global priority of this Alternative with respect to the Goal.

Looking only at Tom, we can see that his priority with respect to the Goal is 0.358, calculated as follows:

- Tom's priority with respect to Experience is 0.217 x 0.547 = 0.119, plus
- Tom's priority with respect to Education is 0.188 x 0.127 = 0.024, plus
- Tom's priority with respect to Charisma is 0.703 x 0.270 = 0.201, plus
- Tom's priority with respect to Age is 0.265 * 0.056 = 0.015,
- For a total of 0.119 + 0.024 + 0.201 + 0.015 = 0.358

Here are the overall priorities for all of the Candidates:

	Priority with Respect to				
Candidate	Experience	Education	Charisma	Age	Goal
Tom	0.119	0.024	0.201	0.015	0.358
Dick	0.392	0.010	0.052	0.038	0.492
Harry	0.036	0.093	0.017	0.004	0.149
Totals:	0.547	0.127	0.270	0.056	1.000

Making the decision

Based on the Board's choice of decision criteria, on their judgments about the relative importance of each, and on their judgments about each candidate with respect to each of the criteria, Dick, with a priority of 0.492, is by far the most suitable candidate. Tom, with a priority of 0.358, is second, and Harry, at 0.149, is third.

The Board should choose Dick as the company's new leader.

Because they have used the AHP, it is easy for them to trace their thinking and to justify the steps along the way to their decision. If they have second thoughts about the final outcome, they can revisit the process and make changes if appropriate.[Note 10] And if they choose to, they can reveal the details of their process to their consultants and confidants, to the candidates, to the shareholders, or to anyone else who might be concerned with the decision.

References

Notes

1. ^ The form and content of the hierarchy are dependent on the wants and needs of the decision makers. If the consultant's plan, for example, included moving all manufacturing to China, the board may well have added *Chinese Language Skills* as one of its Criteria. Also for example, if they had wanted to do so at any point in the process, they could have included *Technical Education* and *Business Education* as Subcriteria under *Education*, or made any other changes that they thought would be helpful in their deliberations.
2. ^ The priorities on any level add up to 1.000, and they represent relative levels of strength or importance. If the priorities of the Criteria were Education=0.500, Experience=0.250, and Charisma and Age each=0.125, then Education would be twice as important as Experience in reaching the Goal, and four times as important as either Charisma or Age. Experience would be half as important as Education and twice as important as Charisma and Age. The actual priorities, of course, depend on the judgments input by the decision makers.
3. ^ The mathematics underlying the AHP has been presented in many books and papers since the 1980s. A technical discussion of the subject is beyond the scope of this article, but a basic explanation for decision makers appears in Saaty (2008), Chapter 5. A much more detailed exposition is in Saaty (2006), Chapters 1 - 3.
4. ^ For one-time decisions like this one, experienced AHP practitioners commonly work from the bottom up, first evaluating the Alternatives with respect to the Criteria, then evaluating the Criteria with respect to the Goal. Where the decision is a repetitive one, they commonly work from the top down.
5. ^ A detailed description of the AHP matrix, including what numbers should go into which cells and why they should go there, can be found in Saaty (2008), Chapter 5. For this example, just follow the colors and everything will end up in the right place.
6. ^ This example uses the Superdecisions AHP software throughout. Like most AHP software, it calculates an Inconsistency Factor, which is a measure of the internal consistency of the judgments entered into the matrix. It is desirable that this factor be less than 0.100. The Inconsistency Factor is further discussed in Saaty (2006), Chapter 2, especially on page 61.
7. ^ An important aspect of the AHP is that it forces the decision makers to look closely at every aspect of the decision.
8. ^ The number of comparisons increases very rapidly with the number of nodes compared. If C is the number of comparisons required and N is the number of nodes to be compared, **C = N x (N-1)/2**. The practical limit on nodes that can be compared is about eight (requiring 8 x 7 / 2 = 28 comparisons). There are other reasons to keep the number of compared nodes below ten: Psychological studies show that the number of items that can be successfully held in mind for activities like this is between five and nine, and some of the math on which the AHP is based is most dependable in this range (see The Magical Number Seven, Plus or Minus Two; Saaty (2006) p. 61; and Saaty (2010), p. 109).
9. ^ In our example, we can easily make these calculations by hand or in a spreadsheet. It can be impractical to do this in real world cases with much larger hierarchies: it's tedious to make so many calculations, and it's easy to introduce errors. Specialized AHP software programs make

these calculations automatically and with a high degree of accuracy.

10. ^ Most AHP software includes features to help in reviewing decisions. One common "what if?" feature is Sensitivity Analysis. This provides an easy way to see, usually graphically, what would happen to the final priorities if a given criterion were given a different weight. The Superdecisions software used in this example revealed that moderate changes in the weight of any criterion would have little effect on the final decision.

Bibliography

The Tom, Dick, and Harry example draws heavily on principles described in the books below, which are the current basic texts on the AHP. They contain detailed descriptions of the theory underlying the process, plus many examples of its use in the real world.

- Saaty, Thomas L. (2006). *Fundamentals of Decision Making and Priority Theory*. Pittsburgh, Pennsylvania: RWS Publications. ISBN 0-9620317-6-3.
- Saaty, Thomas L. (2008). *Decision Making for Leaders: The Analytic Hierarchy Process for Decisions in a Complex World*. Pittsburgh, Pennsylvania: RWS Publications. ISBN 0-9620317-8-X.
- Saaty, Thomas L. (2010). *Principia Mathematica Decernendi: Mathematical Principles of Decision Making*. Pittsburgh, Pennsylvania: RWS Publications. ISBN 978-1-888603-10-1.